TEST III

Math 232
April 22, 2003

Name: _____

By writing my name I swear by the honor code.

Read all of the following information before starting the exam:

- Show all work, clearly and in order. You will not get full credit if I cannot see how you arrived at your answer (even if your final answer is correct).
- Make sure that you follow the directions in each problem and that your answer matches what is asked for.
- Justify your answers algebraically whenever possible. For most problems, work done by calculator will not receive any points (although you may use your calculator to check your answers).
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 8 problems and is worth 100 points, plus some extra credit at the end. Make sure that you have all of the pages!
- Good luck!

1. (*14 pts*) Determine whether each of the following statements is true (T) or false (F).

a. (*2 pts*) **T** **F** $\displaystyle\int_{-2}^{5} (x+2)^3\,dx$ is a real number.

b. (*2 pts*) **T** **F** $\displaystyle\int \frac{1}{x^2+1}\,dx = \frac{1}{2x}\ln|x^2+1| + C.$

c. (*2 pts*) **T** **F** If $u = x^2+1$, then $\displaystyle\int \sqrt{x^2+1}\,dx = \int \sqrt{u}\,du.$

d. (*2 pts*) **T** **F** $\displaystyle\int_0^3 xe^x\,dx = xe^x - \int_0^3 e^x\,dx.$

e. (*2 pts*) **T** **F** $\displaystyle\int \sin^2 x\,dx = \tfrac{1}{2}\int (1-\cos 2x)\,dx.$

f. (*2 pts*) **T** **F** $\displaystyle\int \sin x\,dx = -\cos x + C.$

g. (*2 pts*) **T** **F** An approximation of the arc length of $f(x) = \sin x$ on $[0, \pi]$ with six line segments will be an underapproximation.

2. (*16 pts*) Short answer and fill-ins.

a. (*8 pts*) State the Fundamental Theorem of Calculus.

b. (*8 pts*) The definite integral of a continuous function $f(x)$ from $x = a$ to $x = b$ is defined to be:

$$\int_a^b f(x)\,dx := \lim_{\square} \sum_{\square}^{\square} \square$$

where $\Delta x =$ _____ , $x_k =$ _____ , and $x_k^* \in$ _____ .

3. (*10 pts*) Use integration techniques to find the *exact* value of $\displaystyle\int_0^{\frac{\pi^2}{4}} \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$.

(Don't touch your calculator, even at the end of your calculation. Do all work by hand and show your work carefully.)

4. (*10 pts*) Prove that $\displaystyle\int \sec x\,dx = \ln|\sec x + \tan x| + C$.

5. (*2 pts*) Write your favorite math symbol in this box: $\boxed{}$

6. (*12 pts*) Fill in each blank with one of the seven choices below. (You *might* have to use some choices more than once; in other words, some parts might have the same answer.)

Choices: $f(b) - f(a)$ $f'(b) - f'(a)$ $f(x) - f(a)$ $f'(x) - f'(a)$ $f(x)$ $f'(x)$ 0

$\int_a^b f'(x)\,dx$ = _____ $\frac{d}{dx}\int_a^x f(t)\,dt$ = _____

$\frac{d}{dx}\int_a^b f(x)\,dx$ = _____ $\frac{d}{dx}\int f(x)\,dx$ = _____

7. (*16 pts*) Represent each of the following in terms of integrals.

DON'T SOLVE THE INTEGRALS, JUST WRITE THEM DOWN.

a. (*4 pts*) The "true" area between the graph of $f(x) = x^2 - 4$ and the x-axis on $[0, 3]$ (counting all area positively).

b. (*4 pts*) The average value of $f(x) = x^2 - 4$ on $[0, 3]$.

c. (*4 pts*) The arc length of $f(x) = x^2 - 4$ on $[0, 3]$.

d. (*4 pts*) The function whose value at x is the area under the graph of $f(x) = x^2 - 4$ from 0 to x.

8. (*20 pts*) For each integral given below, *one* of the four integration strategies will work. Circle it.

a. (*4 pts*) $\displaystyle\int \frac{x^3+4}{x^2}\,dx$

I. Substitution with $u = x^3 + 4$

II. Use formulas to get $\frac{\frac{1}{3}x^3+4x}{\frac{1}{4}x^4} + C$

III. Rewrite with algebra, then use formulas

IV. Use formulas to get $\frac{3x^2(x^2)-(x^3+4)(2x)}{x^4}$

b. (*4 pts*) $\displaystyle\int \cos^3 x\,dx$

I. Parts with $u = \cos^3 x$, $dv = dx$

II. Half-angle formulas

III. Pythagorean identity, then $u = \cos x$

IV. Pythagorean identity, then $u = \sin x$

c. (*4 pts*) $\displaystyle\int \frac{\ln 2x}{x^2}\,dx$

I. Subsitution with $u = x^2$

II. Parts with $u = x^2$, $dv = \ln 2x\,dx$

III. Parts with $u = \ln 2x$, $dv = x^2\,dx$

IV. Parts with $u = \ln 2x$, $dv = x^{-2}\,dx$

d. (*4 pts*) $\displaystyle\int \csc^4 x \cot^4 x\,dx$

I. Pythagorean identity, then $u = \csc x$

II. Pythagorean identity, then $u = \cot x$

III. Parts with $u = \csc^4 x$, $dv = \cot^4 x\,dx$

IV. Parts with $u = \csc^2 x \cot^4 x$, $dv = \csc^2 x\,dx$

e. (*4 pts*) $\displaystyle\int \sec^3 x\,dx$

I. Pythagorean identity, then $u = \sec x$

II. Pythagorean identity, then $u = \tan x$

III. Parts with $u = \sec x$, $dv = \sec^2 x\,dx$

IV. Parts with $u = \sec^2 x$, $dv = \sec x\,dx$

Survey Questions: *(2 extra credit points)*

Name a question or topic that could have been on this test, but wasn't.

How do you think you did?

SPACE FOR SCRAP WORK