TEST II

Math 232
March 18, 2004

Name: _____

By writing my name I swear by the honor code.

Read all of the following information before starting the exam:

- Show all work, clearly and in order. You will not get full credit if I cannot see how you arrived at your answer (even if your final answer is correct).
- Make sure that you follow the directions in each problem and that your answer matches what is asked for.
- Justify your answers algebraically whenever possible. For most problems, work done by calculator will not receive any points (although you may use your calculator to check your answers).
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- By writing your name above, you agree to the JMU honor code. In particular, this means that you may not use any notes or crib sheets during this exam, that all work must be your own, and that you may not obtain advance information revealing the problems on this exam.
- This test has 8 problems and is worth 100 points, plus some extra credit at the end. Make sure that you have all of the pages!
- Good luck!

1. (*12 pts*) Determine whether each of the following statements is true (T) or false (F).

(**a**) **T** **F** For all real numbers x we have $1 + \cot^2 x = \csc^2 x$.

(**b**) **T** **F** If θ is an integer multiple of π, then $\cos\theta = -1$.

(**c**) **T** **F** The period of $f(x) = \tan x$ is π.

(**d**) **T** **F** If $\sec x = 2$ then $\cos^{-1} x = \frac{1}{2}$.

(**e**) **T** **F** A limit of the form $\infty - \infty$ is in an indeterminate form.

(**f**) **T** **F** A limit of the form 0^{∞} is in an indeterminate form.

2. (*10 pts*) Give short answers.

(**a**) Give a formal definition of $\sin\theta$, for any angle θ. Your definition should include the words "unit circle," "standard position," "terminal edge," and "coordinate." You can illustrate your definition with a picture if it makes it clearer.

(**b**) Sketch a clear, labeled graph of $f(x) = \sec^{-1} x$. (It may help to begin by sketching the graph of the restricted secant function; if you do this, make sure it is clear which graph is which.)

3. (*10 pts*) Prove that $\frac{d}{dx}(\ln x) = \frac{1}{x}$. (Hint: Start with the fact that $e^{\ln x} = x$ for all $x > 0$.)

4. (*10 pts*) Prove that $\frac{d}{dx}(\cot x) = -\csc^2 x$. (Hint: Use the derivatives of $\sin x$ and $\cos x$.)

5. **(*24 pts*)** Fill in the blanks.

(**a**) $\lim_{x\to 0^+} \log_3 x =$ _____

(**b**) $\frac{d}{dx}(\ln|x|) =$ _____

(**c**) If $\lim_{x\to 0} \ln(f(x)) = -\infty$, then $\lim_{x\to 0} f(x) =$ _____

(**d**) If θ is an angle in standard position whose terminal edge intersects the unit circle at the point $(-\frac{1}{4}, \frac{\sqrt{15}}{4})$, then $\tan\theta =$ _____

(**e**) $\lim_{x\to\infty} \frac{\sin x}{e^x} =$ _____

(**f**) $\lim_{x\to\infty} e^x \sin x =$ _____

(**g**) $\lim_{x\to 0^+} \frac{x}{1-\cos x} =$ _____

(**h**) $\lim_{x\to 2} \frac{\sin(x^2-4)}{x^2-4} =$ _____

(**i**) $\lim_{x\to\infty} \arctan x =$ _____

(**j**) $\frac{d}{dx}(\sec^{-1} x) =$ _____

(**k**) Suppose $f(x)$ is a general sine function with period 4π and a minimum at $x = \frac{\pi}{2}$. The first maximum to the right of this minimum occurs at $x =$ _____

(**l**) Two points for writing anything you want, plus two *extra* points for writing the "secret word" from the policy sheet: _____

6. (*10 pts*) Use logarithmic differentiation to find the derivative of $f(x) = x^{\sin x}$. Show your work very carefully, and circle your final answer.

7. (*24 pts*) Fill in the blanks.

(**a**) The domain of $f(x) = \csc x$ is _____

(**b**) The range of $f(x) = \csc x$ is _____

(**c**) The domain of $f(x) = \cos^{-1} x$ is _____

(**d**) The range of $f(x) = \cos^{-1} x$ is _____

(**e**) $\sin(\sin^{-1} x) = x$ for all x in the interval _____

(**f**) The graph of the function $f(x) = -4\sin(3(x - \pi)) + 2$ has:

amplitude: _____ period: _____ center point: _____

(**g**) If $\sin^{-1} x = \theta$ and $\sin\theta$ is negative, then θ is in the _____ quadrant.

(**h**) List all the x-values of the inflection points of $f(x) = \cos x$: _____

Survey Questions: *(2 extra credit points)*

Name a question or topic that could have been on this test, but wasn't.

How do you think you did?

SPACE FOR SCRAP WORK