

# TEST III

Math 232  
April 22, 2004

Name: \_\_\_\_\_  
By writing my name I swear by the honor code.

**Read all of the following information before starting the exam:**

- Show all work, clearly and in order. You will not get full credit if I cannot see how you arrived at your answer (even if your final answer is correct).
- Make sure that you follow the directions in each problem and that your answer matches what is asked for.
- Justify your answers algebraically whenever possible. For most problems, work done by calculator will not receive any points (although you may use your calculator to check your answers).
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- By writing your name above, you agree to the JMU honor code. In particular, this means that you may not use any notes or crib sheets during this exam, that all work must be your own, and that you may not obtain advance information revealing the problems on this exam.
- This test has 7 problems and is worth 100 points, plus some extra credit at the end. Make sure that you have all of the pages!
- Good luck!

1. (14 pts) Determine whether each of the following statements is true (T) or false (F).

(a)     **T**   **F**      $\left(\sum_{k=1}^n \frac{1}{k+1}\right) \left(\sum_{k=1}^n k^5\right) = \sum_{k=1}^n \frac{k^5}{k+1}.$

(b)     **T**   **F**     If  $\sum_{k=1}^4 a_k = 7$ ,  $\sum_{k=0}^4 b_k = 10$ , and  $a_0 = 2$ , then  $\sum_{k=0}^4 (2a_k + 3b_k) = 48.$

(c)     **T**   **F**      $\cot(\sin^{-1} x) = \frac{\sqrt{1-x^2}}{x}.$

(d)     **T**   **F**     If  $u = x^2$  then  $\int \cos(x^2) dx = \int \cos u du.$

(e)     **T**   **F**      $\int \frac{1}{x^3+1} dx = \frac{1}{3x^2} \ln|x^3+1|.$

(f)     **T**   **F**      $\int xe^x(x+2) dx = x^2e^x + C.$

(g)     **T**   **F**     I would like two free points, please.

2. (12 pts) Give short answers.

(a)     State the Fundamental Theorem of Calculus. (Remember that a theorem is always of the form “If . . . , then . . . .”)

(b)     Write down the general Riemann sum for approximating the area under a function  $f(x)$  from  $x = a$  to  $x = b$  using the Midpoint Rule. Be sure to define what you mean by “ $\Delta x$ ” and “ $x_k$ ”.

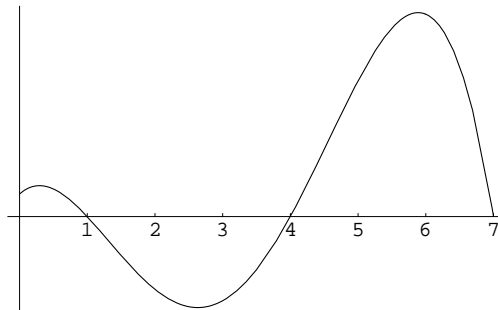
**3.** (16 pts) Each question below asks you to do PART of an approximation problem. Please pay close attention to the directions and answer only what is asked for.

(a) If you were to approximate the area under the function  $f(x) = x + 1$  from  $x = 1$  to  $x = 3$  by using the Left Hand Sum and 4 rectangles, what would be the area of the third rectangle in your approximation?

(b) Suppose now that you want to approximate the arc length of the function  $f(x) = x + 1$  from  $x = 1$  to  $x = 3$  by using 4 line segments. What is the length of the third line segment in your approximation?

**4.** (8 pts) Let  $f$  be the function shown below, and define  $A(x) = \int_0^x f(t) dt$ . In the blanks provided below, list the following quantities in order from smallest to largest:

- A.  $A(0)$
- B.  $A(3)$
- C.  $A(6)$
- D.  $A(7)$



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5. (10 pts) Solve  $\int \left( e^{3x} - \frac{1}{4\sqrt{x}} + \tan(1 - 2x) \right) dx$ , using whatever methods you like.

6. (16 pts) In the following problems you will describe HOW to solve some integrals. PLEASE DO NOT SOLVE THE INTEGRALS. Just fill in the blanks.

(a) The integral  $\int \frac{e^x}{e^x - 1} dx$  can be solved using integration by substitution. What should you choose for  $u$ ?

$$u = \underline{\hspace{2cm}}$$

(b) The integral  $\int \cos^5 x \sin^4 x dx$  can be solved by using Pythagorean identities to write the integral in a form that is ready for a  $u$ -substitution. What will the eventual  $u$ -substitution be?

$$u = \underline{\hspace{2cm}}$$

(c) The integral  $\int x \ln x dx$  can be solved using integration by parts. What should you choose for  $u$  and  $dv$ ?

$$u = \underline{\hspace{2cm}} \quad dv = \underline{\hspace{2cm}}$$

(d) The integral  $\int \frac{1}{\sqrt{4 - 9x^2}} dx$  can be solved using trigonometric substitution. What should the substitution be?

$$\underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

7. (24 pts) Write down definite integrals to represent each of the following quantities.  
**PLEASE DO NOT SOLVE THE INTEGRALS, just write them down.**

- (a) The signed area between the graph of  $y = 3 - x$  and the  $x$ -axis from  $x = 0$  to  $x = 4$ .

- (b) The true area between the graph of  $y = 3 - x$  and the  $x$ -axis from  $x = 0$  to  $x = 4$ .

- (c) The area between the graphs of  $y = 3 - x$  and  $y = 5$  from  $x = 0$  to  $x = 4$ .

- (d) The average value of the function  $y = 3 - x$  from  $x = 0$  to  $x = 4$ .

- (e) The arc length of the graph of  $y = 3 - x$  from  $x = 0$  to  $x = 4$ .

- (f) The volume of the solid obtained from rotating the region between  $y = 3 - x$  and the  $x$ -axis from  $x = 0$  to  $x = 2$  around the  $x$ -axis.

- (g) The volume of the solid obtained from rotating the region between  $y = 3 - x$  and the  $x$ -axis from  $x = 0$  to  $x = 2$  around the  $y$ -axis.

- (h) A definite integral that you would definitely not want to solve.

**Survey Questions:** *(2 extra credit points)*

Name a question or topic that could have been on this test, but wasn't.

How do you think you did?

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**SPACE FOR SCRAP WORK**