232 FTC Quiz
April 6, 2008

Name: * key *

By printing my name I pledge to uphold the honor code.

You may NOT use your notebook or any other materials.

1. State each of the following. Remember to include hypotheses of theorems and definitions of any notation you use.

Give the Riemann Sum definition of $\int_{-1}^{2} x^3\,dx$.
DO NOT SIMPLIFY OR COMPUTE.

4 pts

for example, $x_k^* = x_k = -1 + \frac{3k}{n}$ if RHS.

$$\lim_{n\to\infty} \sum_{k=1}^{n} (x_k^*)^3 \cdot \frac{3}{n}$$

2 pts here
2 pts defs

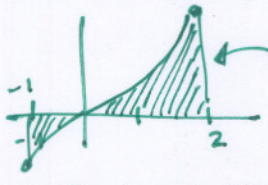
$a = -1,\ b = 2 \qquad x_k = -1 + \frac{3k}{n}$

$\Delta x = \frac{2-(-1)}{n} = \frac{3}{n} \qquad x_k^* \in [x_{k-1}, x_k]$

8 pts

What does the quantity above measure? Feel free to draw a picture.

2 pts



the area between $f(x) = x^3$ and x-axis on $[-1, 2]$

Use the Fundamental Theorem of Calculus to compute this quantity.

2 pts

$$\int_{-1}^{2} x^3\,dx = \left[\frac{x^4}{4}\right]_{-1}^{2} = \frac{2^4}{4} - \frac{(-1)^4}{4} = \frac{16}{4} - \frac{1}{4} = \boxed{\frac{15}{4}}.$$

told class: in each case a deriv. and an integral are applied in some sense. fill in blanks w/o int. signs.

2. Fill in the blanks to answer each question below.

In what sense is the indefinite integral the "opposite" of differentiation (or not)?

$$\int f'(x)\,dx = \underline{\quad f(x) + C \quad} \qquad \frac{d}{dx}\int f(x)\,dx = \underline{\quad f(x) \quad}$$

1 pt f(x) | 1 pt f(x)+C

In what sense is the definite integral the "opposite" of differentiation (or not)?

$$\int_a^b f'(x)\,dx = \underline{\quad f(b) - f(a) \quad} \qquad \frac{d}{dx}\int_a^b f(x)\,dx = \underline{\quad 0 \quad}$$

1 pt f(b)-f(a)

2 pts each → 12 pts

In what sense are area accumulation functions the "opposite" of differentiation (or not)?

$$\int_a^x f'(t)\,dt = \underline{\quad f(x) - f(a) \quad} \qquad \frac{d}{dx}\int_a^x f(t)\,dt = \underline{\quad f(x) \quad}$$

1 pt f(x)-f(a)

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