

Name: _____

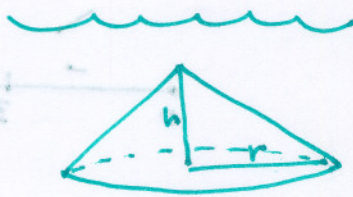
** key **

By printing my name I pledge to uphold the honor code.

You may use your class notebook as long as all writing in the notebook is your own.
No books, photocopied material, extra papers, calculators, cell phones, or peeking.

1. **Word Problem.** An underwater volcano begins to rise up from the seafloor, in water that is 100 meters deep. The volcano always forms a perfect cone, with radius exactly twice its height. If the height of the volcano grows at a rate of 3 meters per year, how fast is material being added to the volcano at the instant the cone breaks the water's surface? For full credit you must justify your answer and show your work carefully and in order.

exactly like #27 Sxn 7.4 (diff. #'s)



$$r = 2h \Rightarrow \frac{dr}{dt} = 2 \frac{dh}{dt} = 2 \cdot 3 = 6$$

$$\frac{dh}{dt} = 3$$

$$\frac{dV}{dt} \Big|_{h=100} = ?$$

10 pts

$$V = \frac{1}{3} \pi r^2 h$$

$$\frac{dV}{dt} = \frac{2}{3} \pi r \frac{dr}{dt} h + \frac{1}{3} \pi r^2 \frac{dh}{dt}$$

$$\frac{dV}{dt} \Big|_{h=100} = \frac{2}{3} \pi \cdot 200 \cdot 6 \cdot 100 + \frac{1}{3} \pi \cdot 200^2 \cdot 3$$

$$\hookrightarrow = 80,000 \pi + 40,000 \pi = \underline{\underline{120,000 \pi \text{ m}^3/\text{yr}}}$$



-OR-
could write $V = \frac{1}{3} \pi r^2(h) = \frac{1}{3} \pi (2h)(h) = \frac{2}{3} \pi h^3$
and go from there ... that's actually easier.

exactly like #56 $s \times n 7.5$ (but with one # changed to make easier)

2. Curve Sketching. Sketch a careful, labeled graph of $f(x) = x^{\frac{3}{2}}(3x - 15)$, including the locations of any roots, extrema, inflection points, holes, asymptotes, etc. Show all work clearly so I can see how you arrived at your graph. More information means a better graph and a higher score.

$$f(x) = x^{\frac{3}{2}}(3x - 15) = 3x^{\frac{5}{2}} - 15x^{\frac{3}{2}}$$

domain $x \geq 0$

roots $x=0, x=5$

$$f'(x) = \frac{15}{2}x^{\frac{3}{2}} - \frac{45}{2}x^{\frac{1}{2}} = \frac{15}{2}\sqrt{x}(x-3)$$

def'd $x \geq 0$

roots $x=0, x=3$

$$f''(x) = \frac{45}{4}x^{\frac{1}{2}} - \frac{45}{4}x^{-\frac{1}{2}}$$

$$\hookrightarrow = \frac{45}{4}\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) = \frac{45}{4}\left(\frac{x-1}{\sqrt{x}}\right)$$

def'd $x > 0$

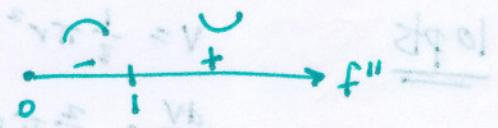
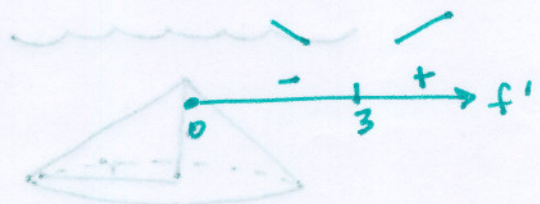
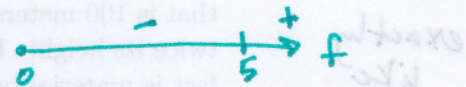
root $x=1$ (DNE $x=0$)

$$f(0) = 0$$

$$f(3) = 3^{\frac{3}{2}}(3 \cdot 3 - 15) = 3 \cdot \sqrt{3}(-6) = -18\sqrt{3}$$

$$f(5) = 0$$

$$f(1) = 1^{\frac{3}{2}}(3 \cdot 1 - 15) = -12$$



10 pts

