# 232 TAKE-HOME REVIEW TEST

- You may use your own book, notes, homework, quizzes, and exams.

- You may NOT collaborate with any living being.

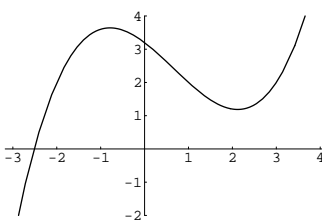- Take the exam in a test-like environment and keep track of how long it takes you.

Math 232w
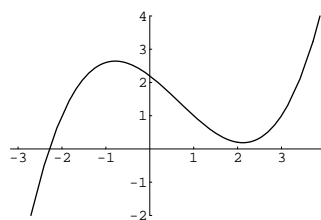January 12, 2008 **Name:** _____

1. **Transformations.** Given the graph of $f(x)$ below on the left, which of the remaining two graphs is $2f(x) + 1$, and which is $2(f(x) + 1)$, and why? Be sure to justify your answers.
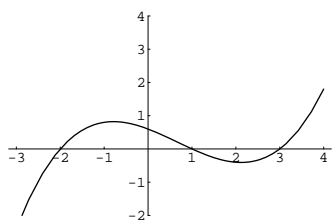
This graph is $f(x)$

Which graph is this, and why?

Which graph is this, and why?

2. **Calculating Limits.** Compute the following limit. Show all of your work.

$$\lim_{x \to 2} \frac{x - 2}{x^3 - 8} =$$

3. **Differentiation.** Use the definition of derivative to differentiate $f(x) = \frac{5}{x}$ in three different ways:

   a) Write $f(x)$ as a power function and differentiate using the power rule.

   b) Using the quotient rule.

   c) Using the definition of derivative (with limits!)

4. **Notation.** Translate the following mathematical sentence into an English sentence. (That is, what would you say if you were to read it out loud? You may read $\mathbb{Q}$ as "fancy Q".)

$$\mathbb{Q} = \{x \in \mathbb{R} \mid x = \tfrac{a}{b} \text{ for some } a, b \text{ integers, where } b \neq 0\}$$

5. **More Notation.** Use the formal definition of $|x|$ to show that $|x|$ is always positive.

6. **Continunity.** Continuous functions have the interesting and useful property that two nearby points in the domain get sent to two nearby points in the range. How is this consistent with the definition of continuity? (WARNING: The definition of a limit should appear somewhere in your answer.)

7. **Optimization.** Alina wants to make a rectangular box with a top that is twice as long as it is wide and is lined (on the inside and outside) with velvet. If she has 240 square inches of velvet, what are the dimensions of the box with the largest volume?

8. **Definition of Limit.** Given the function $f(x)$ shown in the graph below, estimate a positive value of δ for which $0 < |x - 2| < \delta$ guarantees that $|f(x) - 3| < \frac{1}{2}$ and put your answer in the blank. For full credit, draw something on the graph that supports your answer.
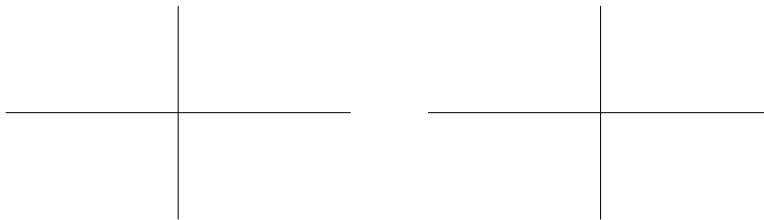
$\delta \approx$ _____
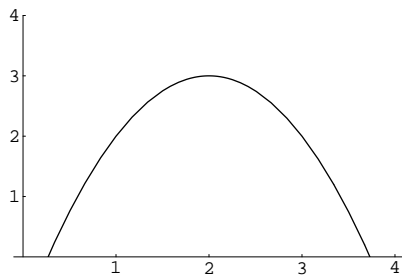
9. **Composition and Inverses.** Let $f(x) = x^{\frac{1}{3}}$.

   a) Find $f^{-1}(x)$.

   b) Use the definition of an inverse function to prove that your inverse from part (a) is correct.

   c) Graph both $y = f(x)$ and $y = f^{-1}(x)$ using the axes below.

   d) How do these graphs compare and why?

10. **Definition of Derivative.** Below is the graph of some function $f(x)$. Sketch the tangent line to $f(x)$ at $x = 2$, and then sketch nearby secant lines from $x = 2$ to $x = 2 + h$ for the following four values: $h = 2$, $h = 1$, $h = 0.5$, and $h = -1$.
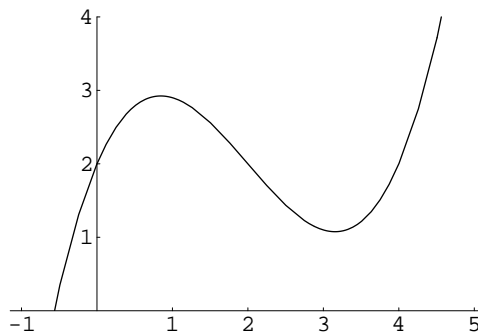
11. **Important Theorems.** Describe in (your own) words and pictures the meanings of each of the following theorems:

a) The Intermediate Value Theorem

b) The Mean Value Theorem

c) The Extreme Value Theorem

12. How long did it take you to complete this test? How do you think you did?