232 TEST 1

You may use your notebook during this exam.
You may NOT use calculators, cell phones, or peeking.

Math 232
February 18, 2009

Name: * key *
By printing my name I pledge to uphold the honor code.

1. Define each of the following in complete sentences:

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3 **a)** e $e = \lim_{h\to 0}(1+h)^{1/h}$ -OR- e is the unique # s.t. $\lim_{h\to 0}\frac{e^h-1}{h} = 1$

3 **b)** $\log_2 x$ $\log_2 x$ is the inverse of the fncn e^x -OR- $\log_2 x$ is the power which you have to raise 2 to, in order to get x

3 **c)** a function f *dominates* a function g
if $\lim_{x\to\infty}\frac{f(x)}{g(x)} = \infty$ (for fncns f and g that $\to\infty$ as $x\to\infty$)

2. Assume you have \$5000 in the bank. Give the formula for your bank balance $Q(t)$ at the end of t years in each of the following conditions:

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3 **a)** Continuous growth at a rate of 4% per year $Q(t) = 5000\,e^{.04t}$

3 **b)** Yearly percentage growth at a rate of 4% $Q(t) = 5000\,(1.04)^t$

3 **c)** Growth at 4% per year compounded monthly $Q(t) = 5000\left(1+\frac{.04}{12}\right)^{12t}$

3. Simplify each of the following as much as possible.

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3 **a)** $\frac{\ln 32}{\ln 2} = \log_2 32 = 5$ $(2^5 = 32)$

3 **b)** $2\log_2 3 - \log_2 5 = \log_2\left(\frac{3^2}{5}\right) = \log_2\left(\frac{9}{5}\right)$

3 **c)** $\log_2 \frac{1}{8} = -3$ $\left(2^{-3} = \frac{1}{8}\right)$

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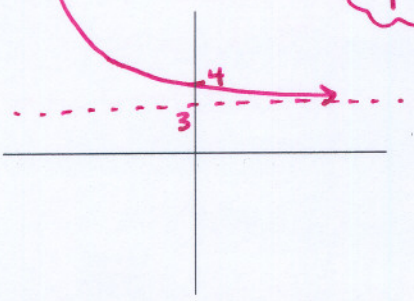
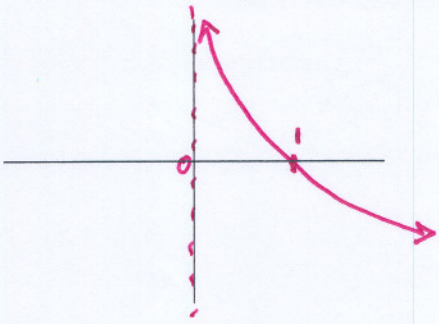
4. Sketch quick graphs of each of the following functions, without using any derivative information. Label all asymptotes and intercepts on the graphs.

12 = 6 each

$f(x) = 2^{-x} + 3$ ($\neq 2^{-x}$)



$f(x) = \ln(\frac{1}{x}) = -\ln x$ ($\neq +\ln x$)



(1/3/6 points each)

5. Calculate each of the following limits and select your answer from A–E. Letters may be used once, more than once, or not used at all.

16 =

4 = A $\lim_{x\to 0} \frac{x^3}{2^x - 1} \to \frac{0}{0}$ (L'H $\lim_{x\to 0} \frac{3x^2}{(\ln 2)2^x}$ L'H $= \lim_{x\to 0} \frac{6x}{(\ln 2)^2 2^x}$ L'H $= \lim_{x\to 0} \frac{6}{(\ln 2)^3 2^x} \to 0$)

4 = A $\lim_{x\to 1^+} (\ln x)^{\frac{1}{x-1}} \to 0^\infty$ ($0^\infty \to 0$)

4 = E $\lim_{x\to\infty} (2^x - 4^x) \to \infty - \infty$ ($= \lim_{x\to\infty} 2^x(1 - 2^x) \to \infty(-\infty)$)

4 = E $\lim_{x\to\infty} \log_{\frac{1}{3}} x \to$ ($\neq \log_{1/3} x$)

A) 0
B) 1
C) e
D) ∞
E) $-\infty$

6. Some of the limits below require multi-step processes to solve. Indicate a viable first step by choosing one of A–C. If a limit could be solved immediately without any of A–C then select D. Letters may be used more than once, or not used at all.

16 =

4 = B $\lim_{x\to\infty} (\frac{1}{2})^x x^5 \to 0 \cdot \infty$ (rewrite as $\frac{x^5}{2^x}$)

4 = D $\lim_{x\to\infty} \frac{2^{-x}}{x^2 + 1} \to \frac{0}{\infty} \to 0$

4 = D $\lim_{x\to\infty} \left(\frac{1}{\ln x}\right)^x \to \frac{1}{\infty^\infty} \to 0$

4 = C $\lim_{x\to\infty} (\ln x)^{\frac{1}{x}} \to \infty^0$ use logs

A) can apply L'Hôpital's Rule immediately
B) need to do algebra before L'Hôpital's Rule
C) must use logarithms
D) could be solved right now

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7. Compute the following derivatives. Show all of your work, but do not simplify your answers! Please put boxes around your final answers.

6 a) $f(x) = 7x^3 e^{2x}$

$$f'(x) = 21x^2 e^{2x} + 7x^3 \cdot 2e^{2x}$$

1/3/5/6 points

6 b) $f(x) = \dfrac{\log_2 x}{x^2+1}$

$$f'(x) = \frac{\frac{1}{(\ln 2)x}(x^2+1) - \log_2 x\,(2x)}{(x^2+1)^2}$$

1/3/4/5/6

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6 c) $f(x) = (\ln 2)\ln(\ln x)$

$$f'(x) = (\ln 2)\frac{1}{\ln x}\cdot\frac{1}{x}$$

1/3/4/5/6

6 d) $f(x) = x^x$

$$\ln(f(x)) = \ln(x^x) = x\ln x$$

$$\frac{1}{f(x)} f'(x) = 1\cdot\ln x + x\cdot\frac{1}{x}$$

$$f'(x) = x^x(\ln x + 1)$$

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Confidential Survey:
We may be switching groups after this exam. Please answer the following questions. Your answers will not be revealed to your other groupmates.

Your name: _____ Your group's name: _____

Please circle one: **A)** Please don't break up our group!

B) Either way is okay with me

C) I would prefer to try working with another group

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