232 TEST 2

You may use your notebook during this exam.
You may NOT use calculators, cell phones, or peeking.

Math 232
March 20, 2009

Name: * key *

By printing my name I pledge to uphold the honor code.

1. Calculate each of the following. Show your work.

12 pts

a) $\sum_{k=1}^{100} \frac{k^2-1}{4} = \frac{1}{4}\left(\sum_{k=1}^{100} k^2 - \sum_{k=1}^{100} 1\right) = \boxed{\frac{1}{4}\left(\frac{100(101)(201)}{6} - 100\right)}$ (84,487.5)

-1 pts 1/4 bad
-4 pts $\sum 1 = 1$
3 pts write out only

12 pts

b) $\sec^{-1}(-2) = \theta$

$\sec\theta = -2$

$\theta \in (0, \pi/2) \cup (\pi/2, \pi)$

$\cos\theta = -\frac{1}{2}$

$\rightarrow \boxed{\theta = \frac{2\pi}{3}}$

4 pts knowing that $\cos\theta = -1/2$

12 pts

c) $\frac{d}{dx}\left(\ln(\sin^2 x)\right) = \boxed{\frac{1}{\sin^2 x} \cdot 2\sin x \cdot \cos x}$ $(2\cot x)$

-4 one lost chain
-2 one extra thing
2 pity, 4 something good

12 pts

d) $\lim_{x\to 0} \frac{\tan x \cos x}{x^2 \csc x} \rightarrow \frac{0 \cdot 1}{0 \cdot \frac{1}{0}} \rightarrow ?$

$= \lim_{x\to 0} \frac{\frac{\sin x}{\cos x} \cdot \cos x}{x^2 \cdot \frac{1}{\sin x}} = \lim_{x\to 0} \frac{\sin^2 x}{x^2} = \lim_{x\to 0} \left(\frac{\sin x}{x}\right)^2 = 1^2 = \boxed{1}$

(OR L'H: $\lim_{x\to 0} \frac{2\sin x\cos x}{2x} \overset{L'H}{=} \lim_{x\to 0} \frac{2\cos^2 x + 2\sin^2 x}{2}$ $= \frac{2}{2} = 1$.)

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-1 missing lims
8 pts to $\frac{\sin^2 x}{x^2}$ or correct L'H twice

-1-4 L'H right away
-4 to L'H after simp.

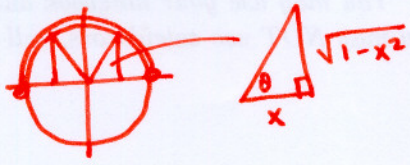
2. Write $\sin(\cos^{-1} x)$ as an algebraic function. Show your work.

12 pts

let $\theta = \cos^{-1} x$

so $\cos\theta = x$

$\theta \in [0, \pi]$



So $\sin(\cos^{-1} x) = \sin\theta = \boxed{\sqrt{1-x^2}}$, rest. to $[-1, 1]$.

(same in 2nd quad)

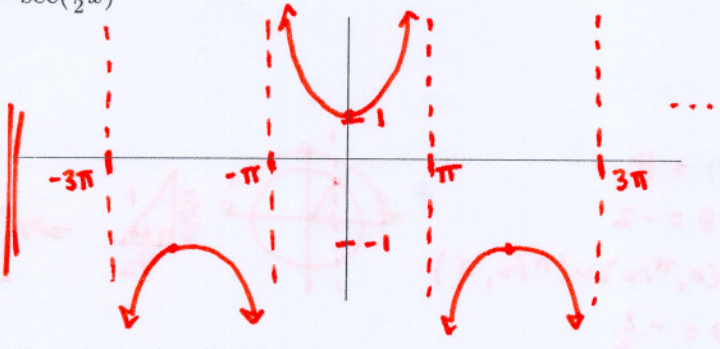
10 pts 1st quad only but $[0, \pi]$ indicated
9 pts other quad bad/gone

3. Sketch graphs of each of the following functions, without using any derivative information. Label your axes carefully.

a) $f(x) = \sec(\frac{1}{2}x)$

12 pts

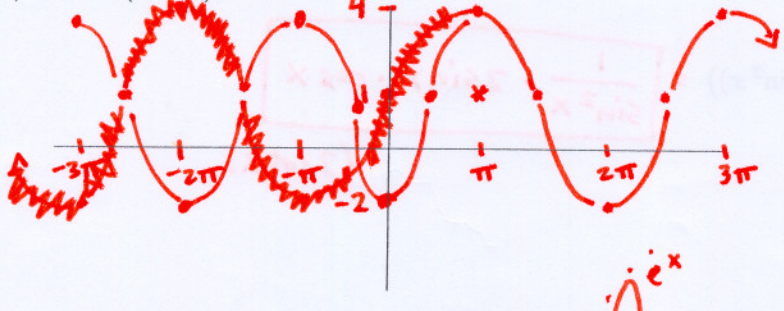
sec 2x - 8
sec x - 4
½ sec x - 6
sec ¼x - 6



(sec x stretched ↔ x 2, so period 2π)

b) $f(x) = 3\cos(x - \pi) + 1$

12 pts

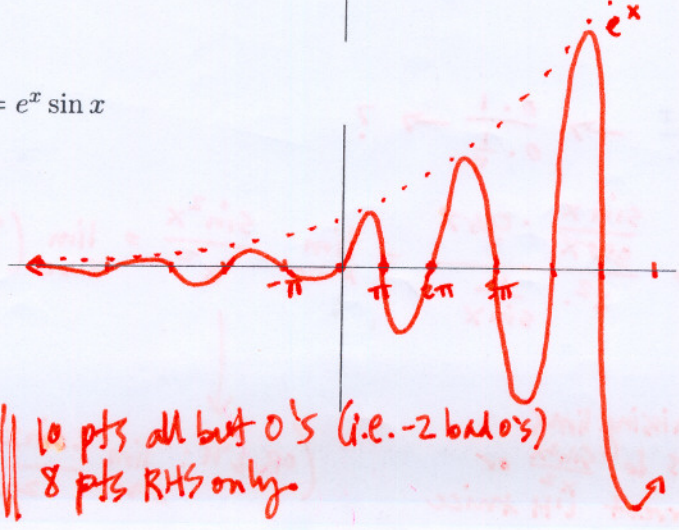


(amplitude 3, period 2π, "center" $(\pi, 1)$)

4 pts each

c) $f(x) = e^x \sin x$

12 pts



(oscillates as $x \to \pm\infty$ but smaller on L and bigger on R; zeros still at $k\pi$)

10 pts all but 0's (i.e. -2 bad 0's)
8 pts RHS only

+4 free

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