232 TEST 3

You may use your notebook during this exam.
You may NOT use calculators, cell phones, or peeking.

Math 232
April 14, 2009

Name: * key *

By printing my name I pledge to uphold the honor code.

1. Compute $\int_3^6 x^2\,dx$ exactly using a limit of Riemann sums.

(According to FTC, your final answer should be 63.)

$$\Delta x = \frac{6-3}{n} = \frac{3}{n}$$

$$x_k^* = x_k = 3 + k\cdot\Delta x = 3 + \frac{3k}{n} \quad \text{(RHS)}$$

$$\lim_{n\to\infty}\sum_{k=1}^{n} f(x_k^*)\,\Delta x = \lim_{n\to\infty}\sum_{k=1}^{n}\left(3+\frac{3k}{n}\right)^2\cdot\frac{3}{n}$$ ← plug into formula 10 pts

$$= \lim_{n\to\infty}\sum_{k=1}^{n}\left(9+\frac{18k}{n}+\frac{9k^2}{n^2}\right)\cdot\frac{3}{n}$$

40 pts

$$= \lim_{n\to\infty}\left(\frac{27}{n}\sum_{k=1}^{n}1 + \frac{18\cdot 3}{n^2}\sum_{k=1}^{n}k + \frac{27}{n^3}\sum_{k=1}^{n}k^2\right)$$ ← algebra & separate the sums 10 pts

$$= \lim_{n\to\infty}\left(\frac{27}{n}\cdot n + \frac{18\cdot 3}{n^2}\cdot\frac{n(n+1)}{2} + \frac{27}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}\right)$$ ← sum formulas 10 pts

$$= \lim_{n\to\infty}\left(27 + \frac{54n^2+\cdots}{2n^2} + \frac{27\cdot 2n^3+\cdots}{6n^3}\right)$$

$$= 27 + \frac{54}{2} + \frac{27\cdot 2}{6} = 27+27+9 = \underline{\underline{63}}.$$ ← calculate limit 10 pts

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2. Explain how the Mean Value Theorem is used in the proof of the Fundamental Theorem of Calculus.

MVT allows us to choose a point x_k^* in each subinterval $[x_{k-1}, x_k]$ at which $F'(x_k^*) = \dfrac{F(x_k) - F(x_{k-1})}{x_k - x_{k-1}}$.

15 pts

2pts sigh
3/2 upside-down : −2
"2/3 + C" : −2
+/− only error : −1
no $\cos\frac{\pi}{2}$ value : −1
F(a) − F(b) : −2

3. Compute the following definite/indefinite integrals.

(typo)

a) $\displaystyle\int_0^{\pi/2} \sqrt{\cos x}\,\sin x\,dx$

(2 pts ① choice of u
3 pts ② successful u-sub
5 pts ③ integrate
5 pts ④ plug in/FTC

① $\left[\begin{array}{l} u = \cos x \\ du = -\sin x\,dx \end{array}\right]$

15 pts

$$-\int_{x=0}^{x=\pi/2} \sqrt{u}\,du = -\int_{x=0}^{x=\pi/2} u^{1/2}\,du = \left[-\frac{1}{3/2}u^{3/2}\right]_{x=0}^{x=\pi/2} = -\frac{2}{3}\left(\sqrt{\cos\frac{\pi}{2}}\right)^3 + \frac{2}{3}\left(\sqrt{\cos 0}\right)^3$$

② ③ ④

$$= -\frac{2}{3}\left(\sqrt{0}\right)^3 + \frac{2}{3}\left(\sqrt{1}\right)^3 = \frac{2}{3}.$$

b) $\displaystyle\int \frac{x}{e^x}\,dx = \int x e^{-x}\,dx$

2 pts ugh
1 pt pity

ln|e^x| : −4 pts
−4 each bad int.
4 pts parts valid attempt

$\left[\begin{array}{l} u = x \longrightarrow du = dx \\ v = -e^{-x} \longleftarrow dv = e^{-x}\,dx \end{array}\right]$

15 pts

2pts

$$-xe^{-x} + \int e^{-x}\,dx = -xe^{-x} + (-e^{-x}) + C = -xe^{-x} - e^{-x} + C.$$

c) $\displaystyle\int \tan^3 x\,dx = \int \tan^2 x \tan x\,dx = \int (\sec^2 x - 1)\tan x\,dx$

3 pts use id
2 pts split
then 5/5

$$= \int \tan x \sec^2 x\,dx - \int \tan x\,dx$$

15 pts

$\left[\begin{array}{l} u = \tan x \\ du = \sec^2 x\,dx \end{array}\right]$ $\left[\begin{array}{l} w = \cos x \\ dw = -\sin x\,dx \end{array}\right]$

$\left\{\begin{array}{l} \dfrac{\sin^2 x}{\cos^2 x} + \dfrac{\cos^2 x}{\cos^2 x} = \dfrac{1}{\cos^2 x} \\ \tan^2 x + 1 = \sec^2 x \end{array}\right.$

$$= \int u\,du + \int \frac{1}{w}\,dw$$

$$= \frac{u^2}{2} + \ln|w| + C = \frac{\tan^2 x}{2} + \ln|\cos x| + C.$$

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