

232 Quiz 2

September 9, 2011

Name: _____ * Key * VI

Section: _____

Name: _____

Work with your partner on each problem; do not split up problems or tasks. You must discuss each problem together and agree on a final solution. Hand in one quiz per group.

You may use your hand-written Notebooks but no other materials and no technology at all. Please keep your discussions quiet so as not to disturb or inform other groups.

1. If a quantity quadruples every 12 years, what is its yearly percentage growth rate? Show all work clearly and in order.

$$Q(12) = 4Q_0 \quad \text{and} \quad Q(t) = Q_0(1+r)^t$$

$$\Rightarrow Q_0(1+r)^{12} = 4Q_0 \quad *$$

$$\Rightarrow 1+r = \sqrt[12]{4}$$

$$\Rightarrow r = \boxed{\sqrt[12]{4} - 1}$$

$$\left(\begin{array}{l} * \text{ or: } 12 \ln(1+r) = \ln 4 \\ \ln(1+r) = \frac{\ln 4}{12} \\ 1+r = e^{\ln 4 / 12} \end{array} \right)$$

note $k = \frac{\ln 4}{12}$ is the continuous growth rate; starting w/ this and converting to yearly gives yearly rate $r = e^{\frac{\ln 4}{12}} - 1$ (equivalent)

2. Use implicit differentiation and the fact that $\log_3 x$ is the inverse of 3^x to prove that $\frac{d}{dx}(\log_3 x) = \frac{1}{(\ln 3)x}$. Make sure that all of your steps are clear.

$$3^{\log_3 x} = x \quad \text{for all } x \quad (\text{def. of inverses})$$

$$\Rightarrow \frac{d}{dx}(3^{\log_3 x}) = \frac{d}{dx}(x)$$

$$\Rightarrow (\ln 3) 3^{\log_3 x} \cdot \frac{d}{dx}(\log_3 x) = 1$$

$$\Rightarrow \frac{d}{dx}(\log_3 x) = \frac{1}{(\ln 3) 3^{\log_3 x}} = \frac{1}{(\ln 3) x} \quad \square$$