232 Quiz 2

September 9, 2011.

Name: _____ *key* _____ v2

Section: _____   Name: _____

Work with your partner on each problem; do not split up problems or tasks. You must discuss each problem together and agree on a final solution. Hand in one quiz per group.

You may use your hand-written Notebooks but no other materials and no technology at all. Please keep your discussions quiet so as not to disturb or inform other groups.

1. If a quantity <u>triples</u> every 15 years, what is its <u>yearly</u> percentage growth rate? Show all work clearly and in order.

$$Q(15) = 3Q_0 \quad \text{and} \quad Q(t) = Q_0(1+r)^t$$

$$\Rightarrow Q_0(1+r)^{15} = 3Q_0 \quad \text{\#}$$

$$\Rightarrow 1+r = \sqrt[15]{3}$$

$$\Rightarrow \boxed{r = \sqrt[15]{3} - 1}$$

note $k = \frac{\ln 3}{15}$ is the continuous growth rate; starting w/this and converting to yearly gives yearly rate $r = e^{\frac{\ln 3}{15}} - 1$ (equivalent)

\# or: $15\ln(1+r) = \ln 3$

$\ln(1+r) = \frac{\ln 3}{15}$

$1+r = e^{\ln 3/15}$

2. Use implicit differentiation and the fact that $\log_2 x$ is the inverse of 2^x to prove that $\frac{d}{dx}(\log_2 x) = \frac{1}{(\ln 2)x}$. Make sure that all of your steps are clear.

$$2^{\log_2 x} = x \quad \text{for all } x \quad (\text{def. of inverses})$$

$$\Rightarrow \frac{d}{dx}\left(2^{\log_2 x}\right) = \frac{d}{dx}(x)$$

$$\Rightarrow (\ln 2)\, 2^{\log_2 x} \cdot \frac{d}{dx}(\log_2 x) = 1$$

$$\Rightarrow \frac{d}{dx}(\log_2 x) = \frac{1}{(\ln 2)\, 2^{\log_2 x}} = \frac{1}{(\ln 2)\, x} \cdot \quad \square$$