232 TEST 2

You may use your notebook during the last fifteen minutes of this exam.
You may NOT use calculators, cell phones, loose papers, or peeking.

Math 232
October 20, 2011   Name: _____ ✻ key ✻ _____

1. For each problem CIRCLE all that apply, and CROSS OUT all that do not apply. If you do nothing on an item I will assume that you meant to cross it out. Problems could possibly have one, many, or no answers to circle.

3 pts each on/off

   a) Which of the following are true statements?

   $\sec^{-1} x = \cos x$ ✗ $\sec^{-1} x = \dfrac{1}{\cos^{-1} x}$ ✗ $\boxed{\sin^{-1} \tfrac{1}{2} = \tfrac{\pi}{6}}$ $\tan^{-1} 0$ is undefined ✗

   (12 poss.)

   circle none in Sxn o1

   b) Which of the following is a 40-rectangle Right Sum approximation for $\int_2^6 f(x)\, dx$?

   $\displaystyle\sum_{k=1}^{40} f(2+\tfrac{k}{100})(0.1)$ ✗ $\boxed{\displaystyle\sum_{k=1}^{40} f(2+\tfrac{k}{10})(0.1)}$ $\displaystyle\sum_{k=1}^{40} f(2+\tfrac{k}{100})(0.01)$ ✗ $\displaystyle\sum_{k=1}^{40} f(2+\tfrac{k}{10})(0.01)$ ✗

   (12 poss.)

   $\Delta x = \dfrac{6-2}{40} = \dfrac{4}{40} = \dfrac{1}{10} = 0.1$

   $x_k = a + k\Delta x = 2 + 0.1k = 2 + \dfrac{k}{10}$

   c) Which of the following can we integrate using the techniques covered so far in class?

   $\boxed{\displaystyle\int \frac{1}{1+x}\, dx}$ $\displaystyle\int \frac{x}{1+x}\, dx$ ✗ $\displaystyle\int \frac{x^2}{1+x}\, dx$ ✗ $\boxed{\displaystyle\int \frac{1}{1+x^2}\, dx}$ $\boxed{\displaystyle\int \frac{x}{1+x^2}\, dx}$

   (15 poss.) $\ln|1+x| + C$ $\tan^{-1} x + C$ $\tfrac{1}{2}\ln|1+x^2| + C$

   d) Which of the following are equivalent to the Fundamental Theorem of Calculus?

   $\left[\int f'(x)\, dx\right]_a^b = \int_a^b f(x)\, dx$ ✗ $\int_a^b G(x)\, dx = [G'(x)]_a^b$ ✗ $\boxed{\displaystyle\int_a^b h''(x)\, dx = [h'(x)]_a^b}$

   (9 poss.)

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2. Write down the n-rectangle Right Sum approximation for $\int_2^4 (x^2 + 1)\, dx$ and simplify until that the only letters that appear in the sum are k and n.
Please do NOT try to calculate the sum; just write it down.

13 pts

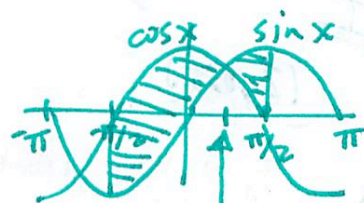$$\Delta x = \frac{4-2}{n} = \frac{2}{n}$$

$$x_k^* = 2 + k \cdot \frac{2}{n}$$

$$\sum_{k=1}^{n} \left(\left(2 + \frac{2k}{n}\right)^2 + 1 \right)\left(\frac{2}{n}\right)$$

3. In each problem write the quantity described in terms of one or more definite integrals.
Please do NOT try to solve the definite integrals; just write them down.

a) The area between the graphs of $\sin x$ and $\cos x$ on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

13 pts

$$\int_{-\pi/2}^{\pi/4} (\cos x - \sin x)\, dx + \int_{\pi/4}^{\pi/2} (\sin x - \cos x)\, dx$$
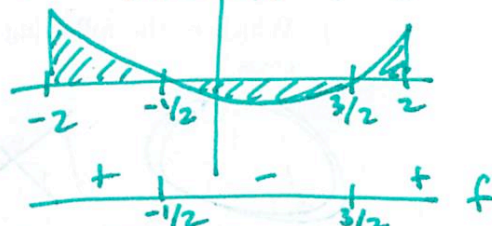
$\sin x = \cos x$ at $x = \frac{\pi}{4}$

b) The unsigned area of the region between the graph of $f(x) = 4x^2 - 4x - 3$ and the x-axis on the interval $[-2, 2]$.

13 pts

$$\int_{-2}^{-1/2} f(x)\, dx - \int_{-1/2}^{3/2} f(x)\, dx + \int_{3/2}^{2} f(x)\, dx$$

$4x^2 - 4x - 3 = 0$

$(2x-3)(2x+1) = 0$

c) The change in temperature, after 10 minutes, of a hot potato that is cooling at a rate of $T'(t) = -15e^{-0.5t}$ degrees per minute.

13 pts

$$\int_0^{10} -15e^{-0.5t}\, dt$$

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