

232 Quiz 5

Name: \_\_\_\_\_

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Name: \* Key \*

Name: \_\_\_\_\_

Work in groups. You must do all problems together, discussing and agreeing on your answer. You may not "split up" the work. You may use your Notebooks.

Solve each of the following integrals. Show your RELEVANT work clearly and in order.

1.  $\int \frac{x}{\sin^2 x} dx = \int x \csc^2 x dx$  (like #65 from 9.2)

$$\left[ \begin{array}{l} u=x \rightarrow du=dx \\ v=-\cot x \leftarrow dv=\csc^2 x dx \end{array} \right]$$

$$= -x \cot x + \int \cot x dx$$

$$= -x \cot x + \int \frac{\cos x}{\sin x} dx \quad \left[ \begin{array}{l} u=\sin x \\ du=\cos x dx \end{array} \right]$$

$$= -x \cot x + \int \frac{1}{u} du = \boxed{-x \cot x + \ln |\sin x| + C}$$

2.  $\int \frac{3x^2 + x + 1}{(x^2 + 1)^2} dx$  (like #36 from 9.3)

$$\text{PF: } \frac{x^2 + x + 1}{(x^2 + 1)^2} = \frac{Ax + B}{x^2 + 1} + \frac{Cx + D}{(x^2 + 1)^2}$$

$$x^2 + x + 1 = (Ax + B)(x^2 + 1) + (Cx + D)$$

$$x^2 + x + 1 = (A)x^3 + (B)x^2 + (A+C)x + (B+D)$$

$$\begin{cases} A=0 & A+C=1 \\ B=1 & B+D=1 \end{cases} \rightarrow \begin{cases} A=0 & C=1 \\ B=1 & D=0 \end{cases}$$

$$= \int \frac{1}{x^2 + 1} dx + \int \frac{x}{(x^2 + 1)^2} dx \quad \left[ \begin{array}{l} u=x^2 + 1 \\ du=2x dx \\ \frac{1}{2} du = dx \end{array} \right]$$

$$= \tan^{-1} x + \frac{1}{2} \int \frac{1}{u^2} du = \boxed{\tan^{-1} x + \frac{1}{2} \left( \frac{1}{-1} (x^2 + 1)^{-1} \right) + C}$$