out of 71

232 EXAM 2

You may use your notebook during the last 10 minutes of this exam.

You may NOT use calculators, cell phones, loose papers, or peeking.

Math 232
March 18, 2014

Name: * key *

By printing my name I pledge to uphold the honor code.

1. Determine whether each of the following is True (T) or False (F).

2 each
6 pts

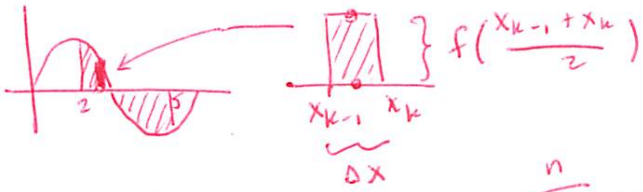
(T) F $\int_0^{\frac{3\pi}{2}} \cos x \, dx$ is negative.

T (F) The proof of the Fundamental Theorem of Calculus involves something called a "lighthouse" sum. (telescoping)

(T) F The proof of the Fundamental Theorem of Calculus hinges on the Mean Value Theorem.

2. Use sigma notation to express the n-rectangle Midpoint Sum approximation of the signed area between the graph of $f(x) = \sin x$ and the x-axis on $[2, 5]$. Simplify your expression until the only letters that appear are n and k, but do NOT attempt to evaluate the sum. Show key steps in your work. *(like 7.2 #28)*

8 pts



$a=2, b=5$

$\Delta x = \frac{5-2}{n} = \frac{3}{n}$

$x_k = a + k\Delta x = 2 + \frac{3k}{n}$

$$\sum_{k=1}^{n} \sin\left(\frac{\left(2+\frac{3(k-1)}{n}\right)+\left(2+\frac{3k}{n}\right)}{2}\right)\left(\frac{3}{n}\right)$$

3. Circle ALL of the following that are equivalent to the Fundamental Theorem of Calculus, and cross out the rest. *(like 7.5 #16,17)*

2 each
8 pts

~~A)~~ $\int_a^b H(x)\,dx = [H'(x)]_a^b$ $((H')' \neq H)$

~~C)~~ If $f'(x) = F(x)$, then $\int_a^b f(x)\,dx = [F(x)]_a^b$. $(F' \neq f)$

(B) $\int_a^b g''(x)\,dx = [g'(x)]_a^b$ $((g')' = g'')$

(D) $\int_a^b f(x)\,dx = g(b) - g(a)$, if $g'(x) = f(x)$.

$(g' = f)$

FTC says
$\int_a^b f(x)\,dx = F(b) - F(a)$ if $F' = f$
or in other words
$\int_a^b f'(x)\,dx = f(b) - f(a)$
(since $f' = f'$ ☺)

22

333
6767
50
338350

101
67
707
606
6767

4. Calculate each of the following and put your answers in the blanks. Show key steps of your work to the right of your answers. *(like 7.1 #35, 7.4 #27,42, 7.5 #21,52, 7.6 #28)*

5 each
35 pts

$\sum_{k=1}^{100}(3-k^2) =$ $\underline{-338,050}$

$\sum_{k=1}^{100} 3 - \sum_{k=1}^{100} k^2 = 3(100) - \frac{100(101)(201)}{6}$ (50, 67, 3, 1)

$= 300 - 338,350$

$\int_1^4 2(3^x)\,dx =$ $\underline{\frac{2\cdot 81}{\ln 3} - \frac{2\cdot 3}{\ln 3}}$ (156/ln 3)

$\left[2\frac{1}{\ln 3}3^x\right]_1^4 = 2\cdot\frac{3^4}{\ln 3} - 2\cdot\frac{3^1}{\ln 3}$

$\int_{-1}^{3}(x^2-1)\,dx =$ $\underline{16/3}$

$\left[\frac{x^3}{3} - x\right]_{-1}^{3} = \left(\frac{3^3}{3} - 3\right) - \left(\frac{(-1)^3}{3} - (-1)\right)$

$6 - (-2/3) = 16/3$

$\int_{-1}^{3}|x^2-1|\,dx =$ $\underline{8}$

split at $x=1$

$-(-2/3) + 2/3 + 6 - (-2/3) = 8$

$\int \frac{3x+1}{\sqrt{x}} =$ $\underline{2x^{3/2} + 2x^{1/2} + C}$

$\int 3x^{1/2}\,dx + \int x^{-1/2}\,dx = 3\left(\frac{1}{3/2}\right)x^{3/2} + \left(\frac{1}{1/2}\right)x^{1/2} + C$

$\int \frac{2}{1+9x^2} =$ $\underline{\frac{2}{3}\tan^{-1}(3x) + C}$

guess: $\frac{d}{dx}\left(\frac{2}{3}\tan^{-1}(3x)\right) = \frac{2/3}{1+(3x)^2}(3)$ ✓

$\int_2^3 \frac{\ln x}{x}\,dx =$ $\underline{\frac{(\ln 3)^2}{2} - \frac{(\ln 2)^2}{2}}$

hard!

guess: (note $\frac{d}{dx}(\ln x) = 1/x$)

$\frac{d}{dx}\left(\frac{(\ln x)^2}{2}\right) = 2\left(\frac{\ln x}{2}\right)^1\left(\frac{1}{x}\right)$ ✓

$\left[\frac{(\ln x)^2}{2}\right]_2^3 = \frac{(\ln 3)^2}{2} - \frac{(\ln 2)^2}{2}$

5. Briefly explain each of the following in one or two sentences. Please be concise.

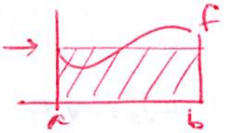
4 each
12 pts

a) How do you find the limit of a rational function as $x \to \infty$?

if deg of numerator is > deg of denom, then lim is $\pm\infty$
if deg (num) < deg (denom), then lim is 0
if deg (num) = deg (denom), then lim is ratio of leading coefficients.

b) How do you find the average value of a function $f(x)$ on an interval $[a,b]$?

$\frac{1}{b-a}\int_a^b f(x)\,dx$



c) What is the meaning of the expression x_k^* in the context of this class?

$x_k^* \in [x_{k-1}, x_k]$ is a point we choose in the k^{th} subinterval.

47