The simplest and most straightforward type of proof is a “direct” proof, which we’ll call any proof that follows straight from the definitions or from a direct calculation. Here’s a couple of examples:

First we’ll prove the following statement:

*The sum of any two rational numbers is rational.*

This proof follows directly from the definition of what it means for a number to be rational.

**Given that:** $r$ and $s$ are rational numbers.

**Show that:** $r + s$ is rational.

**Proof:** Since $r$ and $s$ are rational, we can write $r = p/q$ and $s = m/n$ for some integers $p$, $q$, $m$, and $n$.

Then $r + s = \frac{p}{q} + \frac{m}{n} = \frac{pm + qm}{qn}$.

Since $p$, $q$, $m$, and $n$ are integers, $pm + qm$ and $qn$ are also integers (the sum or product of integers is an integer). Thus by the calculation above, $r + s$ is the quotient of two integers, and is therefore a rational number. ■

Here’s an example of a proof that is really just a calculation. Given the trigonometric identity $\sin(x + y) = \sin x \cos y + \cos x \sin y$, we’ll prove the identity:

$$\sin(2x) = 2 \sin x \cos x$$

**Given that:** $\sin(x + y) = \sin x \cos y + \cos x \sin y$.

**Show that:** $\sin(2x) = 2 \sin x \cos x$.

**Proof:**

$$\sin(2x) = \sin(x + x)$$
$$= \sin x \cos x + \cos x \sin x \quad \text{(by the sum identity)}$$
$$= 2 \sin x \cos x. \quad \blacksquare$$

Note that this proof is merely a string of equalities connecting $\sin(2x)$ to $2 \sin x \cos x$. Many proofs are like this; when proving an identity or an equation we often start from one side of the equation and work towards the other.