Direct Proof

Math 235 Fall 2000

The simplest and most straightforward type of proof is a "direct" proof, which we'll call any proof that follows straight from the definitions or from a direct calculation. Here's a couple of examples:

First we'll prove the following statement:

The sum of any two rational numbers is rational.

This proof follows directly from the definition of what it means for a number to be rational.

Given that: r and s are rational numbers.

Show that: $r + s$ is rational.

Proof: Since r and s are rational, we can write $r = p/q$ and $s = m/n$ for some integers p, q, m, and n.

Then $r + s = \dfrac{p}{q} + \dfrac{m}{n} = \dfrac{pn + qm}{qn}$.

Since p, q, m, and n are integers, $pn + qm$ and qn are also integers (the sum or product of integers is an integer). Thus by the calculation above, $r + s$ is the quotient of two integers, and is therefore a rational number. ■

Here's an example of a proof that is really just a calculation. Given the trigonometric identity $\sin(x + y) = \sin x \cos y + \cos x \sin y$, we'll prove the identity:

$$\sin(2x) = 2 \sin x \cos x$$

Given that: $\sin(x + y) = \sin x \cos y + \cos x \sin y$.

Show that: $\sin(2x) = 2 \sin x \cos x$.

Proof:

$$\begin{aligned} \sin(2x) &= \sin(x + x) \\ &= \sin x \cos x + \cos x \sin x \quad \text{(by the sum identity)} \\ &= 2 \sin x \cos x. \end{aligned}$$ ■

Note that this proof is merely a string of equalities connecting $\sin(2x)$ to $2 \sin x \cos x$. Many proofs are like this; when proving an identity or an equation we often start from one side of the equation and work towards the other.