

Using Function Information

Math 235 Fall 2000

1. Consider the following information about a function $f(x)$:

- $f(x)$ is continuous everywhere except $x = 1$.
- $f(x)$ is differentiable everywhere except at $x = -2$, $x = 1$, and $x = 3$.
- $\lim_{x \rightarrow -2^-} f'(x) = -\infty$, $\lim_{x \rightarrow -2^+} f'(x) = -\infty$.
- $\lim_{x \rightarrow 1^-} f(x) = \infty$, $\lim_{x \rightarrow 1^+} f(x) = -\infty$.
- $\lim_{x \rightarrow 3^-} f'(x) = -\infty$, $\lim_{x \rightarrow 3^+} f'(x) = \infty$.

Use this information to find any vertical tangents, cusps, or asymptotes of $f(x)$. After you have done this, sketch a possible graph of $f(x)$.

2. Consider the following information about a function $f(x)$:

- $f(x)$ is continuous and defined on $(0, 8]$.
- $f(x)$ is differentiable everywhere except at $x = 4$ and $x = 6$.
- $f(4) = 3$, $f(6) = 2$, $f(8) = 4$.
- $\lim_{x \rightarrow 0^+} f(x) = 1$.
- $x = 2$ is the only zero of $f'(x)$.
- $f'(1) = 8$, $f'(3) = 4$, $f'(5) = -2$, $f'(7) = 1$.

Find all local and global extrema of $f(x)$ on the interval $(0, 8]$. After you have done this, sketch a possible graph of $f(x)$.

3. Consider the following information about a function $f(x)$:

- $f(x)$ is continuous and defined on $[0, \infty)$.
- $f(x)$ is differentiable everywhere except at $x = 6$.
- $f'(2) = 0$ and $f'(4) = 0$ (and $f'(x)$ has no other roots).
- $f'(3) = 2$, $f'(5) = -2$, and $f'(7) = 3$.
- $f''(2) = 3$ and $f''(5) = -2$.

Find all local minimums and maximums of $f(x)$. What pieces of information did you *not* need to find the local extrema? What other pieces of information do you need to find the global mins and maxes (if any) of $f(x)$ on $[0, \infty)$? Make up values for these other pieces of information (be careful—some values will make the graph impossible to draw given the information above). Given this new information, find any global extrema of $f(x)$ and sketch a possible graph of $f(x)$. What piece of information did you not need at all in this problem?

4. For more practice, do problems #31-34 in 4.4, #29-32 in 4.6, and #43-48 in 4.7.