Chapter 1 Outline

1.2 Notations and Formulas from Elementary Mathematics

- **Real Numbers:** Know classification of real numbers; know and use definitions of rational and irrational numbers; proofs involving rational and irrational numbers; order properties of the real numbers.

- **Set Notation:** Be able to write and decipher set notation; know and use definitions of intersection and union, inclusion, empty set. You should also be able to write sets from future sections in set notation (*e.g.* the graph of a function, the range of a composition, and so on).

- **Inverval Notation:** Be able to express an interval of the real numbers in interval notation (*e.g.* $(4, 6]$) as well as in set notation and as a picture on the real number line.

- **Absolute Value:** Know and use definition of, alternate characterizations of, and properties of the absolute value.

- **Boundedness:** What does it mean for an interval to be bounded above or below (can you write it mathematically)? Determine if and/or where a given interval is bounded.

- **Algebra:** Know and use basic factoring formulas (although you *don't* have to memorize the cubic factoring formulas, you should be aware of them and able to apply them to various problems).

- **Geometry:** Know and be able to use the basic area, perimeter, surface area, and volume formulas.

1.3 Inequalities

- **Simple Inequalities:** Know how to solve inequalities using algebra and examination of the signs of the factors of a product or quotient. What conditions on a product (or quotient) make the product zero? Positive? Negative?

- **Inequalities with Absolute Values:** Use the definition of absolute value to find solution sets for inequalities of the forms $|x| < \delta$, $0 < |x - c| < \delta$, and $|x| > \delta$. Understand interpretation of these inequalities as distances. Solve inequalities of the form $|ax + b| < c$ and $|ax + b| > c$ by splitting into two inequalities without absolute values, and connecting with "AND" or with "OR."

- **Triangle Inequality:** Know this inequality by heart. Be able to use it in proofs involving inequalities.

1.4 Coordinate Plane; Analytic Geometry

- **Distances and Midpoints:** Know how to derive the distance formula for points in the plane (from the distance formula $|a - b|$ for real numbers). Be able to verify that the midpoint formula always works. Memorize and be able to apply these formulae.

- **Lines:** Definition of slope and intercepts; various standard forms of equations for lines; finding parallel and perpendicular lines and points of intersection between lines (and between lines and other objects, like circles).

- **Conic Sections:** Be able to recognize the general formula for a conic section, and be able to translate back and forth between the general formula and the "standard" formulae for each type of conice section (by completing the square or multiplying out). Find centers, radii, axes, and so on for various conic sections. You do not need to memorize the formulae for ellipses and hyperbolae. You also do not need to know just by looking at the general formula $Ax^2 + By^2 + Cx + Dy + E = 0$ which type of conic section it represents.

1.5 Functions

- **Definitions of Function, Domain and Range:** Know the definition of a function by heart. Know what the domain, range, and target of a function are, and be able to identify the domain and range graphically, algebraically, or by examining a written function. Be able to construct examples of functions between given sets (of numbers, or people, or whatever). Be able to construct functions with given domain and range.

- **Graphs of Functions:** Understand what the graph of a function represents, and how to write the set of points that make up the graph in set notation Know what the vertical line test is and how to use it. Why does the vertical line test tell you if your graph is a function? (Use the definition of function.) Know the graphs of basic functions by heart, and be able to use your graphing calculator (with the correct window) to graph more complicated functions.

- **Piecewise-Defined Functions:** Be able to write, decipher, and graph piecewise-defined functions. In future sections, be able to do problems involving piecewise-defined functions.

- **Even and Odd Functions:** Know the definition of what it means for a function to be odd or even (algebraically); know what this means geometrically in terms of symmetry and why. Be able to test whether a given function is odd, even, or neither both algebraically and graphically; prove whether combinations of odd and even functions are odd or even.

- **Applications of Functions:** Be able to construct a function from one variable to another given the relationship between them (*e.g.* the perimeter of a sqare as a function of its area).

1.6 The Elementary Functions

- **Polynomial Functions:** Definition of a polynomial function, degree, various coefficients and terms. General or "global" shape of polynomials of even or odd degree; possible number of roots or turn-around points.

- **Rational Functions:** Definition of a rational function. Location of roots and vertical asymptotes; what happens when the numerator and the denominator have a common factor?

- **Trigonometric Functions:** The unit circle, radians, and angle measure. Definitions of the six trigonometric functions both for acute angles (right triangle definitions) and arbitrary angles (unit circle definition). Calculating exact values of the trig functions for a given acute angle using a right triangle; calculating exact values of the trig functions for a given angle using the unit circle. Be able to explain why the two definitions coincide for acute angles. Graphs of the trig functions (don't have to memorize the ones for $\csc x$, $\sec x$, and $\cot x$). Be able to explain why the graphs look the way they do (using the unit circle definitions). Be able to use trigonometric identities and laws of sines and cosines in proofs and in computations (you should know the most basic identities by heart).

1.7 Combinations of Functions

- **Algebraic Combinations:** Definitions of sum, difference, product, quotient, scalar multiple, and linear combination of functions; using this definition to compute such algebraic combinations of functions; domains of such combinations; graphs of such combinations.

- **Composition:** Definition of composition and using it to compute various compositions of functions. Domain of a composition given the domains of the component functions; properties of a composition given the properties of the component functions; "de"composing functions.

- **Transformations:** Given a function $f(x)$, know how the constants A, B, C, and D in the function $Af(B(x+C))+D$ effect the graph of the function (be sure to do this for trig functions and non-trig functions).

1.8 Mathematical Proofs and Proof by Induction

- **Proofs in General:** See the handouts for help on this. You will be required to write proofs on the exam. Know the difference between hypotheses and conclusions; be sure to make your argument logically clear. Know how to do direct proofs and proof by contradiction.

- **Induction:** What is mathematical induction? Know the general form of an inductive proof. When do you use such a proof? Why does such a proof prove a fact for all positive integers? Know not only *how* to write an inductive proof, but *why* it works. Be sure that you can do inductive proofs involving inequalities (possibly with hints) as well as equalities.