Chapter 3 Outline **Math 235 Fall 2000**

3.1 The Derivative

- **Definition of Derivative:** State the formal definition of the derivative (as the limit of slopes of secant lines). Sketch a picture of what this definition means (including labelling what $f(x+h)$, h, and so on are in the picture). Explain in your own words what is happening here. Interpret the derivative at a point as an instantaneous rate of change, *i.e.* the limit of average rates of change over smaller and smaller intervals. What is the definition of a left- or right-derivative? Also make sure you can do the above using the alternate formulation of the definition of derivative given in (3.1.5) on page 127.

- **Calculating Derivatives using the Definition:** Be able to calculate $f'(x)$ directly from the limit definition of derivative (*i.e.* without using any differentiation "rules") either at a point or as a function. Be sure you can do the algebra involved in such calculations!

- **Tangent Lines and Normal Lines:** Know how to use the derivative at a point to find the equations of the tangent and normal lines to the curve at that point. Don't just memorize the formulae in the book for these lines; be able to explain where those formulae came from!

- **Differentiability and Continuity:** Know the definition of differentiability (at a point, on an interval, or as a function). If a function is continuous, must it be differentiable? If a function is differentiable, must it be continuous? Be able to come up with examples illustrating various combinations of differentiability and continuity (*e.g.* a function that is continuous but not differentiable). Determine whether a function is differentiable at a point using the definition (including piecewise functions). Since this section is before the "rules" you should know how to do this *without* the rules. On the other hand, you should also know how to check for differentiability (in particular of a piecewise function) using the rules from the next section.

3.2 Some Differentiation Formulas

- **Stating the Differentiation Rules:** Be able to state all of the differentiation rules, in various notations (including Leibniz notation; see Section 3.3).

- **Proving Differentiation Rules using the Definition of Derivative:** Be able to prove the following differentiation rules by using the definition of derivative: the derivative of a constant is zero, the derivative of x (with respect to x) is 1, the sum rule, the constant multiple rule, the product rule (with hints), and the reciprocal rule.

- **Proving Differentiation Rules by Other Means:** Be able to prove the following differentiation rules *without* going back to the definition of derivative: the power rule for positive integer powers (by induction and the product rule), the power rule for negative integer powers (using the previous rule tand the reciprocal rule), the quotient rule (using the product and reciprocal rules).

- **Calculating Derivatives using the Rules:** Be able to calculate various derivatives using the differentiation rules listed above. Make sure that you can do this quickly and efficiently, but *also* make sure that you can do it one step at a time, listing every rule you used in the right order, if asked to.

3.3 The $\frac{d}{dx}$ Notation; Derivatives of Higher Order

- **Leibniz notation:** Be able to work with this notation; in particular make sure that you know how to write, for example, "the derivative of A with respect to r at the point $r = 5$" in this notation. Be able to state all the derivative rules in this notation.

- **Derivatives of Higher Order:** Know the notation (Leibniz and "prime" notation) for higher-order derivatives. Be able to calculate, say, the fifth derivative of a function. How many times can you differentiate x^n before you get zero? What do you get if you differentiate x^n some smaller number of times? Why?

3.4 The Derivative as a Rate of Change

- **AROC and IROC:** Know how to find average and instananeous rates of change. Be able to find the rate of change of a quantity with respect to different variables (*e.g.* Example 2 on page 145), and be able to interpret the meaning and units of these rates of change.

- **Position, Velocity, and Acceleration:** Know the relationship between position, velocity, and acceleration given by the derivative. Be able to solve word problems about such things. What is the definition of speed? When is an object moving on a line speeding up or slowing down and why? Given a position function construct number lines for the signs of velocity and acceleration, and interpret the results.

3.5 The Chain Rule

- **Stating and Proving the Chain Rule:** Be able to state the chain rule in various notations. You are *not* responsible for knowing how to prove the chain rule. Know the chain rule for iterated compositions (see (3.5.5) on page 158) and be able to use it.

- **Calculating using the Chain Rule:** Calculate various derivatives using the chain rule and the differentiation rules you learned before. Be sure that you can do this in various contexts, in various notations. In particular if I give you $y(u)$ and $u(x)$ be able to calculate $y'(x)$ *without* plugging the formula for u into y. Also be able to do problems like #31–#40 on page 161 where you only know a few values of the functions involved.

3.6 Differentiating the Trigonometric Functions

- **Stating and Using the Trigonometric Differentiation Rules:** Know all six rules by heart, and be able to state them. Be able to apply them in various combinations together with the other differentiation rules. In particular make sure that you know what expressions like $\sin^2(x)$ and $\sin(\tan(x))$ mean and how to differentiate them.

- **Proving the Trigonometric Differentiation Rules:** Be able to prove the formulae for the derivatives of $\sin x$ and $\cos x$ by using the definition of derivative (and the two special trig limits you know). Be able to prove the other four formulae *without* going back to the definition of derivative by using the derivatives of $\sin x$ and $\cos x$.

3.7 Implicit Differentiation; Rational Powers

- **Implicit Functions:** What is an implicit function? Give specific examples and explain what makes them implicit rather than explicit functions.

- **Implicit Differentiation:** What is implicit differentiation and when would you use it? Make sure that you can *correctly* use implicit differentiation by using the chain rule as appropriate. Note that it will be very important here to be aware of what variable you are differentiating with respect to, and which other variables are functions of that variable. Of course a lot of the things we did above can be done using implicit differentiation, *e.g.* higher derivatives, slopes or equations of tangent lines, etc.

- **Rational Powers:** Be able to prove the power rule for rational powers using some algebra and implicit differentiation, and the power rule for integer powers. Be able to do this in general (for any power p/q) and in specific examples (*e.g.* for the function $f(x) = x^{3/2}$. Be able to state and prove this rule and use it in various calculations.

3.8 Rates of Change Per Unit Time

- **Rates of Change:** Again be sure that you can interpret a derivative like $\frac{dA}{dr}$ in terms of A and r (one example would be $A =$ area and $r =$ radius). What are the units? What does it mean if $\frac{dA}{dr}$ is positive or negative or zero? What if instead we have $\frac{dA}{dt}$ where t is a time variable? What are the units? What does it mean if $\frac{dA}{dt}$ is positive or negative or zero?Be sure you can use Leibniz notation because "prime" notation will be ambiguous for these sorts of problems.

- **Related Rates Problems:** Be able to translate, set up, and solve related rates problems. Your solution should always include a labeled picture (if appropriate) and clear name assignments for your variables. In the setup of your problems you will get partial credit for clearly stating what rate(s) you know and what rate(s) you are trying to find. When you differentiate you will often have to use the chain rule (do you know why? that's very important). Make sure you do not confuse expressions like $\frac{dA}{dt}$ (which is a function) and $\frac{dA}{dt}|_{t=1}$ (which is a number).