Math 235 Fall 2000
Quiz 4 (9/21) Name: _____ 03/05

You have 20 minutes to take this quiz. Each problem will be graded for clarity of work as well as correctness, so show all work **clearly and in order**. Circle or otherwise indicate your final answers. Please note that there are problems on both the front and the back of this page.

1. (*6 points*) **[Similar to #39, 2.3]**
Calculate the following limit (or write "DNE", if the limit does not exist):

$$\lim_{x\to 4}\left(\left(\frac{1}{x}-\frac{1}{4}\right)\left(\frac{1}{x-4}\right)\right).$$

2. (*6 points*) **[Similar to #33, 2.2]**
Let f be some function for which you know only that:

$$\textit{If } 0<|x-3|<1, \textit{ then } |f(x)-5|<0.1.$$

Which of the following statements are necessarily true? (Circle **T** if the statement is necessarily true, and **F** otherwise.)

(**a**) **T** **F** If $0<|x-3|<\frac{1}{4}$, then $|f(x)-5|<\frac{1}{4}(0.1)$.

(**b**) **T** **F** If $|x-2.5|<0.3$, then $|f(x)-5|<0.1$.

(**c**) **T** **F** If $0<|x-3|<1$, then $|f(x)-4.95|<0.05$.

(**d**) **T** **F** If $0<|x-3|<1$, then $|f(x)-5|<0.2$.

(**e**) **T** **F** If $|x-3|<1$, then $|f(x)-5|<0.1$.

(**f**) **T** **F** $\lim_{x\to 3} f(x)=5$.

Turn over for more...

3. (*8 points*) **[Similar to #45 and #51, 2.3]** Prove or disprove the following statements as instructed.

(**a**) The following statement is <u>true</u>:

If $\lim_{x\to c}(f(x)+g(x))$ *exists, but* $\lim_{x\to c} f(x)$ *does not exist, then* $\lim_{x\to c} g(x)$ *does not exist.*

Prove it. (Hint: try proof by contradiction.)

(**b**) The following statement is <u>false</u>:

$$\textit{If } f(x) < g(x) \textit{ for all } x \neq c, \textit{ then } \lim_{x\to c} f(x) < \lim_{x\to c} g(x).$$

Disprove this statement by finding a counterexample. (In other words, find functions f and g and a number c so that the hypothesis is true but the conclusion is false. You don't have to come up with formulae for f and g; a picture of such an f and g will suffice – but be sure that f, g, and c are clearly labeled.)