TEST I

Math 235
September 28, 2000

Name: _____

Read all of the following information before starting the exam:

- Show all work, clearly and in order. I will take off points if I cannot see how you arrived at your answer (even if your final answer is correct).

- Justify your answers algebraically whenever possible. Work done by calculator will <u>not</u> receive any points (although you may use your calculator to check your answers).

- Circle or otherwise indicate your final answers. Put a circle on the scrap page for 2 points.

- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.

- This test has 7 problems and is worth 100 points, plus some extra credit at the end. It is your responsibility to make sure that you have all of the pages!

- Good luck!

1. (*12 points*) Suppose $f(x)$ is a function whose domain is $[0, \infty)$, and let $g(x) = \dfrac{1}{x-2}$. Write your answers to the questions below in interval notation.

    a. (*6 pts*)   What is the domain of $f + g$?

    b. (*6 pts*)   What is the domain of $f \circ g$?

2. (*14 points*) Consider the limit statement $\lim\limits_{x \to 2}(-2x + 1) = -3$.

    a. (*6 pts*)   Write down the delta-epsilon definition of this limit statement.

    b. (*8 pts*)   Give a careful delta-epsilon proof of this limit statement.

3. (*18 points*) Let $f(x)$ be the function defined by the graph below. Use the graph to determine the sets and quantities described in each part. Write your answers in the blanks provided. (*1 pt each except b, g, i, and j are 2 pts.*)
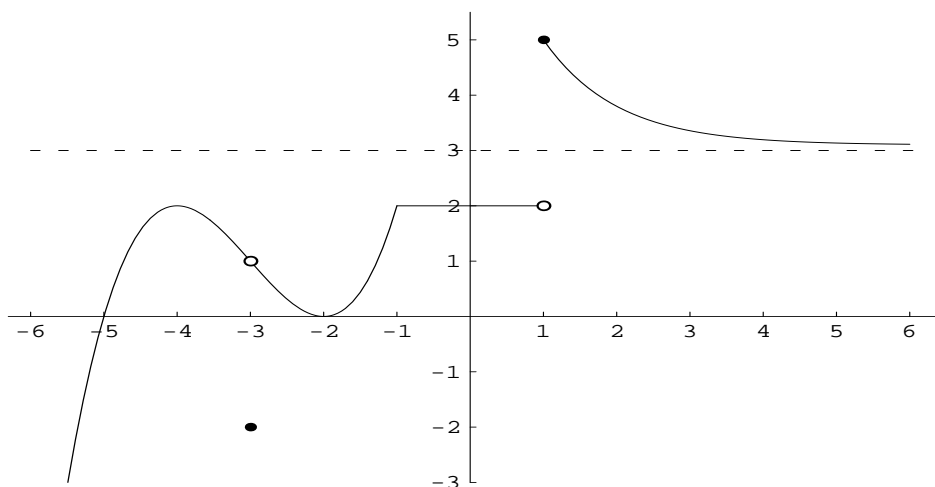
(**a**) _____ The domain of $f(x)$.

(**b**) _____ The range of $f(x)$.

(**c**) _____ $\lim\limits_{x \to -3} f(x)$.

(**d**) _____ $\lim\limits_{x \to 1} f(x)$.

(**e**) _____ $\lim\limits_{x \to -1} f(x)$.

(**f**) _____ $\lim\limits_{x \to 1^-} f(x)$.

(**g**) _____ The solution set of the equation $f(x) = 2$.

(**h**) _____ The solution set of the inequality $f(x) > 2$.

(**i**) _____ List the zeros of $f(x)$.

(**j**) _____ List the zeros of $f(2x)$.

(**k**) _____ List any removable discontinuities of $f(x)$.

(**l**) _____ List any infinite discontinuities of $f(x)$.

(**m**) _____ List any jump discontinuities of $f(x)$.

(**n**) _____ If $g(x) = \frac{1}{x+3}$, what is $g(f(1))$?

4. (*14 points*) Consider the function $f(x) = \dfrac{x^2 + x - 2}{x^2 - x - 6}$. Put your answers into the blanks provided below. (*(a-e) 2 pts each, (f) 4 pts.*)

(a) _____ What is the domain of $f(x)$?

(b) _____ List the roots of $f(x)$.

(c) _____ List the vertical asymptotes of $f(x)$.

(d) _____ What is $\lim\limits_{x \to -2} f(x)$?

(e) _____ At which x-values is $f(x)$ discontinuous?

(f) _____ Solve the inequality $f(x) < 0$. Show your work clearly below.

5. (*12 points*) Suppose $f(x)$ is a function continuous on the interval $[2, 5]$. Answer the questions below as clearly and <u>concisely</u> as you can.

a. (*6 pts*) What does the Extreme Value Theorem tell you about $f(x)$?

b. (*6 pts*) Suppose in addition that $f(2) = 4$ and $f(5) = -1$. What does the Intermediate Value Theorem tell you about $f(x)$?

6. (*16 points*) Consider the function $f(x) = \dfrac{2x}{\tan 3x}$.

a. (*8 pts*)   Check algebraically whether $f(x)$ is an even function, an odd function, or neither. Show your work clearly.

b. (*8 pts*)   Find $\lim\limits_{x \to 0} f(x)$. Show all of your work clearly.

7. (*14 points*) Mark each of the following statements as (always) True (**T**) or as False (**F**). (*2 pts each plus 2 free points for (h).*)

(**a**)   **T**   **F**   If $\theta \in [\pi, \frac{3\pi}{2}]$ with coordinates (x, y) on the unit circle, then $\sin \theta = -y$.

(**b**)   **T**   **F**   Every rational function is a continuous function.

(**c**)   **T**   **F**   Every n-degree polynomial has exactly n real roots.

(**d**)   **T**   **F**   The equality $1 + 3 + 5 + \ldots + (2n + 1) = (n^2 + 2n + 1)$ is true when $n = 1$.

(**e**)   **T**   **F**   If $\lim\limits_{x \to c} f(x) = 0$, then $\lim\limits_{x \to c} (f(x)g(x)) = 0$.

(**f**)   **T**   **F**   The number $5.12112211122211112222\ldots$ is rational.

(**g**)   **T**   **F**   If f and g are continuous at c, then $f \circ g$ is continuous at c.

(**h**)   **T**   **F**   You have done better on this exam than Elvis would have.

Survey Question (2 Extra Credit Points):

How do you think you did on this test?

SCRAP WORK