TEST III

Math 235
December 7, 2000

Name: _____
by writing my name i swear by the honor code

Read all of the following information before starting the exam:

- Show all work, clearly and in order. I will take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible. Work done by calculator will not receive any points (although you may use your calculator to check your answers).
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements, and add two points if you sign your name on the scrap page.
- This test has 8 problems and is worth 100 points, plus some extra credit at the end. It is your responsibility to make sure that you have all of the pages!
- Good luck!

1. (*12 points*) The definite integral $\int_1^3 (9 - x^2)\, dx$ can be approximated using an upper sum with partition $P = \{1, 1.7, 2, 2.4, 3\}$. Assuming such an approximation, find the following values:

(**a**) $\Delta x_3 =$_____ .

(**b**) $x_2^* =$_____ .

(**c**) $M_4 =$_____ .

(**d**) The area of the third rectangle = _____ .

2. (*9 points*) Fill in the blanks by writing each quantity in terms of definite integrals.

DO NOT SOLVE THE INTEGRALS, JUST WRITE THEM DOWN!

(**a**) Rick peels out from a stoplight in his Camero with an acceleration of 20 feet per second per second. If he keeps up this acceleration, his velocity after five seconds is described by the integral:

_____

(**b**) You walk aimlessly along the straight median of a north-south highway in Nevada with velocity $t(4 - t)$ miles per hour. (You can suppose that the "positive" direction here is north, although you won't need that here.) After 5 hours, the total distance you have walked (in any direction) is described by the integral:

_____

(**c**) Consider the situation above (on the median in Nevada). After five hours, your distance from your original starting place is described by the integral:

_____

3. (*20 points*) Calculate the following integrals. One of the integrals in the list is not solvable using the methods of this class. For that integral, write "can't solve."

(**a**) $\displaystyle\int \csc x \cot x \; dx$

(**b**) $\displaystyle\int_1^2 \frac{x^3+5}{x^2}\, dx$

(**c**) $\displaystyle\int_{-2}^2 \frac{x^3}{\sqrt{x^2+1}}\, dx$

(**d**) $\displaystyle\int \cos(x^2)\, dx$

(**e**) $\displaystyle\int \frac{\sin(\sqrt{x})}{\sqrt{x}}\, dx$

4. (*8 points*) Which of the following are anti-derivatives of the function $f(x) = \sin^2 x$? Circle ***all*** that apply.

A. $\frac{1}{3}\sin^3 x$

B. $\frac{1}{3}\sin^3 x \cos x$

C. $2\sin x \cos x$

D. $\dfrac{\sin^3 x}{3\cos x}$

E. $\displaystyle\int_2^4 \sin^2 x \, dx$

F. $\displaystyle\int \sin^2 x \, dx$

G. $\displaystyle\int_3^x \sin^2 t \, dt$

H. $\displaystyle\int_{\cos x}^1 \sin t \, dt$

5. (*12 points*) Fill in the blanks below.

(**a**) $\displaystyle\int y'(u(x))u'(x)\, dx =$ _____ .

(your answer should not have an integral sign in it)

(**b**) $\displaystyle\int (f(x)g'(x) + f'(x)g(x))\, dx =$ _____ .

(your answer should not have an integral sign in it)

(**c**) $\displaystyle\int_0^5 |x^2 - 1|\, dx =$ _____ + _____ .

(split into two integrals that you can solve, but don't solve the integrals)

(**d**) The Mean Value Theorem for Integrals states that if f is a continuous function on $[a, b]$, then:

_____ .

6. (*23 points*) Describe the quantities below in terms of definite integrals. You may get partial credit for drawing a picture or saying a little bit about how you arrived at your integral, but you do not have to show your work to get credit here.

DO NOT SOLVE THE INTEGRALS, JUST WRITE THEM DOWN!

(**a**) The signed area between the graph of $y = x^3 - 8$ and the x-axis on $[0, 5]$.

(**b**) The true area between the graph of $y = x^3 - 8$ and the x-axis on $[0, 5]$.

(**c**) The area of the region bounded by $y = x^3 - 8$, $x = 0$, and $y = 19$.

(**d**) The volume of the solid obtained by rotating the region bounded by $y = x^3 - 8$, $x = 0$, and $y = 19$ about the y-axis.

(**e**) The volume of the solid obtained by rotating the region bounded by $y = x^3 - 8$, $y = -8$, and $x = 3$ about the y-axis, ***using slices***.

(**f**) The volume of the solid obtained by rotating the region bounded by $y = x^3 - 8$, $y = -8$, and $x = 3$ about the y-axis, ***using shells***.

7. (*8 points*) Which of the following are statements of the Fundamental Theorem of Calculus? (There may be more than one answer; choose *all* that apply.)

(**a**) **FTC** **NOT** $\displaystyle\int_a^b g'(x)\,dx = g(b) - g(a)$

(**b**) **FTC** **NOT** $\displaystyle\int_a^b g(x)\,dx = g'(a) - g'(b)$

(**c**) **FTC** **NOT** $\displaystyle\int_a^b F(x)\,dx = f(b) - f(a),$ where $f'(x) = F(x)$

(**d**) **FTC** **NOT** $\displaystyle\int_a^b f(x)\,dx = F(b) - F(a),$ where $f'(x) = F(x)$

8. (*8 points*) Circle True (T) or False (F) as appropriate.

(**a**) **T** **F** $\displaystyle\int_0^1 |\tan(x^3)| \geq \left|\int_0^1 \tan(x^3)\,dx\right|.$

(**b**) **T** **F** $\displaystyle\int_0^1 x^2 f(x)\,dx = \left(\int_0^1 x^2\,dx\right)\left(\int_0^1 f(x)\,dx\right).$

(**c**) **T** **F** If $\displaystyle\int_0^5 f(x)\,dx \geq 0$, then $f(x) \geq 0$ for all x in $[0, 5]$.

(**d**) **T** **F** $\displaystyle\int_1^3 (\tan x + \sec x)\,dx = \int_1^2 (\tan x + \sec x)\,dx + \int_2^3 \tan x\,dx + \int_2^3 \sec x\,dx.$

Survey Question (2 Extra Credit Points):

Which question on this test was the hardest? Which was the easiest?

SCRAP WORK