# TEST II

Math 235
November 9, 2000

**Name:** $\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$
by writing my name i swear by the honor code

**Read all of the following information before starting the exam:**

- Show all work, clearly and in order. I will take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible. Work done by calculator will not receive any points (although you may use your calculator to check your answers).
- Circle or otherwise indicate your final answers. Put a happy face on the scrap page for 2 points.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 9 problems and is worth 100 points, plus some extra credit at the end. It is your responsibility to make sure that you have all of the pages!
- Good luck!

**1.** (*12 points*) Consider the function $f(x) = \dfrac{6}{x+4}$.

(**a**) Use the ***definition of derivative*** to find $f'(2)$. *Note: NO CREDIT will be given if you use any differentiation "rules".*

**2.** (*12 points*) Prove the quotient rule using the product rule and the chain rule. *Do NOT use the definition of derivative here!! Use the product rule and the chain rule!*

**3.** **(*15 points*)** Circle **T** (True) or **F** (False), and fill in the blanks and boxes provided.

(**a**) **T** **F** All continuous functions are differentiable.

(**b**) **T** **F** All differentiable functions are continuous.

(**c**) **T** **F** $f(x) = \dfrac{x^2+1}{\sqrt{x+1}}$ is a rational function.

(**d**) If $\lim_{x\to\infty} f(x) = 2$, then $f(x)$ has a _____ at $y = 2$.

(**e**) If $g(0) = 1$, $g(1) = 2$, $g'(0) = 2$, $g'(1) = 1$, $h(0) = 5$, $h(1) = -2$, $h'(0) = 0$, and $h'(1) = 3$, then $(g \circ h)'(1) =$_____ .

(**f**) If a continuous, differentiable function $f(x)$ has exactly four roots, then its derivative $f'(x)$ has _____ roots.

(**g**) If $f(x)$ is a continuous, differentiable function whose derivative $f'(x)$ has exactly sixteen roots, then the function $f(x)$ has _____ roots.

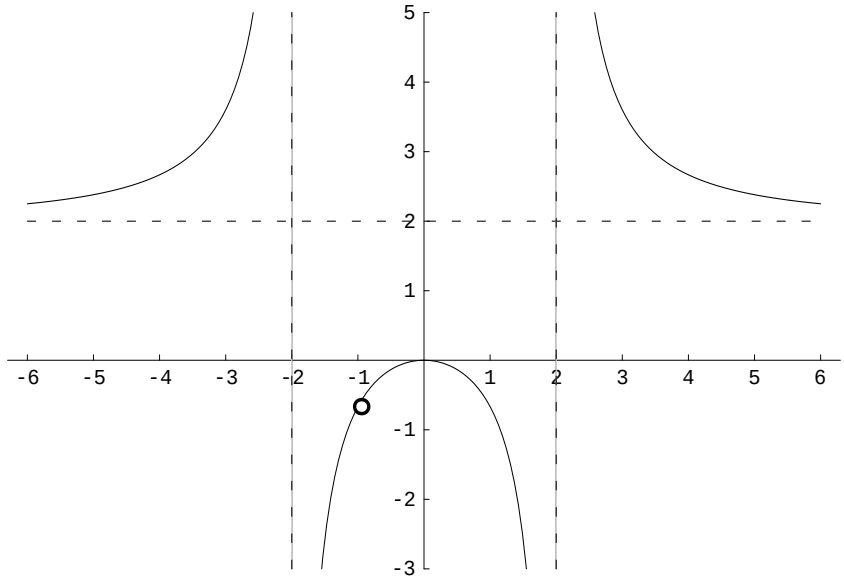
(**h**) A function $f(x)$ is differentiable at $x = 3$ if:

$$\lim_{\square} \frac{\square}{\square} \text{ exists.}$$

(**i**) $\dfrac{f(2+h)-f(2)}{h}$ is the slope of the line from _____ to _____ .

(**j**) Suppose a function $f(x)$ has a local maximum at the point $x = 2$. By definition, this means: _____ . (Hint: your answer should *not* involve the derivative.)

**4.** (*10 points*) Find a rational function that could have the graph below. Be sure to explain how you arrived at your function.




**5.** (*14 points*) Find each of the derivatives below. Show your work clearly and in order.

(**a**) If $y = \csc(\tan(x))$, find $\dfrac{dy}{dx}$.

(**b**) If $rs^2 = 1$ and $s = 3t - 5$, find $\left.\dfrac{dr}{dt}\right|_{s=4}$.

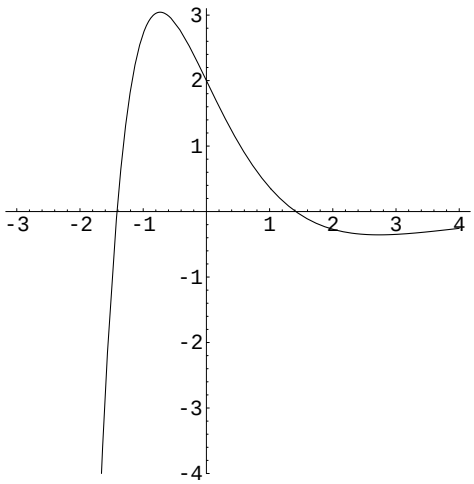
**6.** (*8 points*) Below are the graphs of four functions: the graph of a function $f(x)$, the graph of $f'(x)$, the graph of $f''(x)$, and the graph of some other function, *in no particular order*. Using what you know about derivatives, identify which of the following graphs *must* be $f(x)$, which must be $f'(x)$, and which must be $f''(x)$. Circle the correct answers below.

(**a**) The graph which must be $f(x)$ is: $A$ $B$ $C$ $D$

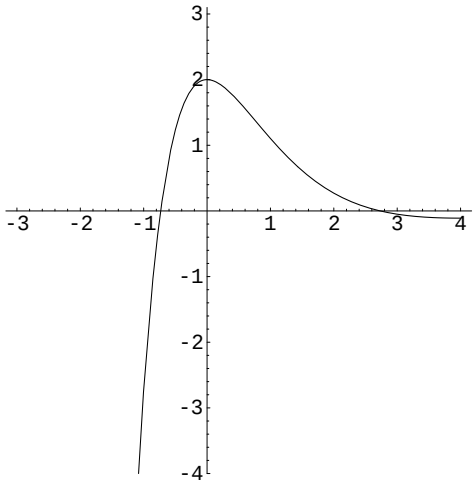
(**b**) The graph which must be $f'(x)$ is: $A$ $B$ $C$ $D$

(**c**) The graph which must be $f''(x)$ is: $A$ $B$ $C$ $D$

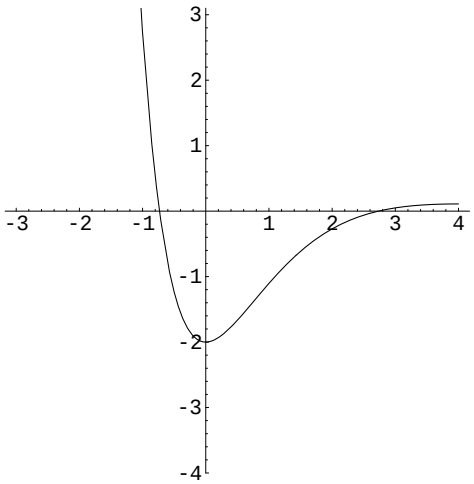
**(A)**



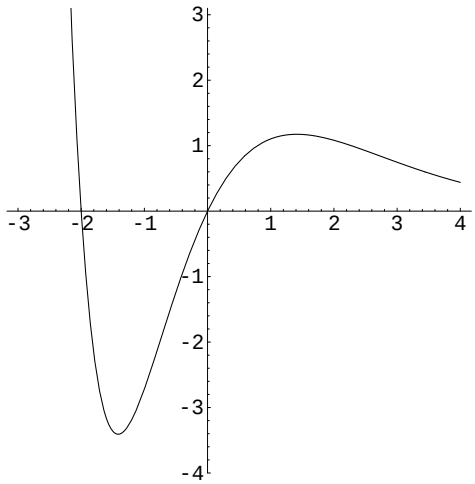

**(B)**




**(C)**




**(D)**



**7.** (*10 points*) Read, but *do not solve*, the following word problem:

> *A poster is to contain 50 square inches of printed matter with margins of 4 inches each at the top and bottom and 2 inches at each side. What overall dimensions, if any, will cause the poster to have a minimum total area?*

Draw a clear, labeled picture for this word problem. Translate this word problem into a math problem ***(but DO NOT SOLVE the problem)*** as follows: Determine the function you are to minimize, write it in terms of ONE variable, and determine the interval we are considering for that variable. Then write your answers in the blanks below. Be sure that any variables you use are clearly marked in your picture. Show any work that you do to get the answers for the blanks below.

The word problem above is equivalent to the following math problem:

Minimize the (one-variable) function: _____

On the interval: _____

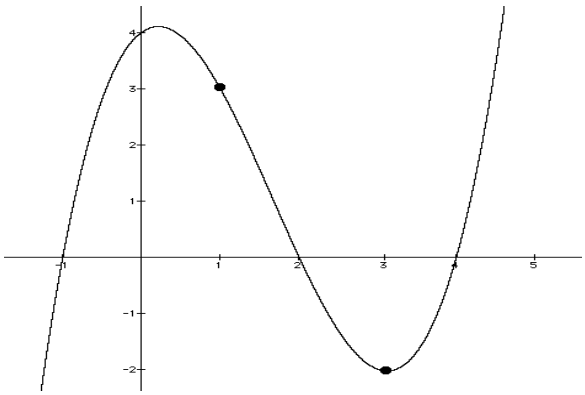
DO NOT SOLVE THE MATH PROBLEM YOU JUST WROTE DOWN

**8.** (*10 points*) Consider the following information about a function $f(x)$:

- $f(x)$ is differentiable everywhere except at $x = 3$.
- $f'(x)$ has exactly two zeros, at $x = 2$ and $x = 6$.
- $f''(1) = 2,\ \ f''(2) = 3,\ \ f''(4) = -2,\ \ f''(6) = -3$.
- $f(5) < f(3)$.

Suppose you want to find the global maximum value of $f(x)$ that occurs on the interval $[0, 5]$. Given the conditions above, list all of the $x$-values at which such a maximum value could possibly occur. (Do not list any values that could *not* produce a global maximum value given the information above.) Justify your answer.

**9.** (*9 points*) Consider the following graph of a function $f(x)$:



Circle *all* of the properties that apply in each of the following:

(**a**) At $x = 1$, $f(x)$ is: *Positive* *Negative* *Zero* *Increasing* *Decreasing*

(**b**) At $x = 1$, $f'(x)$ is: *Positive* *Negative* *Zero* *Increasing* *Decreasing*

(**c**) At $x = 3$, $f'(x)$ is: *Positive* *Negative* *Zero* *Increasing* *Decreasing*

**Survey Question (2 Extra Credit Points):**

How do you think you did on this test?

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**SCRAP WORK**