235 MIDTERM EXAM

You may use your notebook during the last fifteen minutes of this exam.

You may NOT use calculators, cell phones, loose papers, or peeking.

Small numbers to the right of the problems indicate similar homework exercises.

Math 235
November 5, 2012

Name: * key * v1

By printing my name I pledge to uphold the honor code.

2 free

1. Determine whether each of the following is true or false. *#1 all sections*

T (F) If $f'(3) = 0$, then f has a local extremum at $x = 3$.

T (F) The inverse of $y = x^5$ is $y = x^{-5}$.

T (F) $\sec^{-1} x = \frac{1}{\cos^{-1} x}$.

(T) F If f is differentiable on $\mathbb{R}$ and has an extremum at $x = -2$, then $f'(-2) = 0$.

T (F) For all real numbers x, there exists some real number y such that $x = y^2$.

T (F) $\frac{d}{dz}(\frac{1}{z}) = \ln z$.

(T) F If f is an exponential function, then f' is a constant multiple of f.

(T) F $0 < |x - c|$ if and only if $x \neq c$.

(T) F If a circle grows larger in such a way that its radius increases at a constant rate, then its circumference will also increase at a constant rate.

2pts each

18 total

2. Which of the following is equal to the derivative of $f(x) = \sqrt{\ln(x^2+1)}$? *2.5 #36*

(A) $f'(x) = \dfrac{x}{(x^2+1)\sqrt{\ln(x^2+1)}}$

B) $f'(x) = \frac{1}{2}x^{-\frac{1}{2}}\left(\frac{1}{x^2+1}\right)$

C) $f'(x) = (\ln(x^2+1))^{-\frac{1}{2}}(x)$

D) $f'(x) = \frac{1}{2}\left(\frac{2x}{x^2+1}\right)^{-\frac{1}{2}}$

3. What important theorem is a key part of the proof of Rolle's Theorem? *3.1 reading*

A) The Mean Value Theorem

B) The Intermediate Value Theorem

(C) The Extreme Value Theorem

D) The Implicit Function Theorem

4. Which of the following is a function whose derivative is $f'(x) = \dfrac{e^x}{1+e^x}$? 2.5 #63

A) $f(x) = \tan^{-1}(\sqrt{e^x})$

B) $f(x) = \dfrac{e^x}{x+e^x}$

C) $f(x) = \ln(1+e^x)$

D) $f(x) = \dfrac{e^x(1+e^x)+e^x(1+e^x)}{(1+e^x)^2}$

5. Calculate $\lim\limits_{x\to-\infty} \dfrac{e^x}{\tan^{-1} x}$. Ch.1 rev #22

A) $\frac{\pi}{2}$ B) ∞ C) 1 D) 0

6. Which of the four intervals I below is the largest interval on which we can guarantee that $x^2 \in (3.5, 4.5)$ for all $x \in I$? 1.2 #15

A) $(1.9, 2.1)$

B) $(2-3.5, 2+4.5)$

C) $(1.5, 2.5)$

D) $(\sqrt{3.5}, \sqrt{4.5})$

7. Suppose that r is an independent variable, s is a function of r, and q is a constant. Calculate $\frac{d}{dr}(qs+r^2)$. 2.4 #56

A) $q\frac{ds}{dt} + \frac{dq}{dt}s + 2r\frac{dr}{dt}$

B) $q\frac{ds}{dr} + \frac{dq}{dr}s + (\frac{ds}{dr})^2$

C) $\frac{dq}{dr}\frac{ds}{dr} + 2\frac{dr}{dr}$

D) $q\frac{ds}{dr} + 2r$

8. What is the domain of the function $\sqrt{(x-1)(x+2)}$? 1.1 #35

A) $(-2, 1)$

B) $[-2, 1]$

C) $(-\infty, -2) \cup (1, \infty)$

D) $(-\infty, -2] \cup [1, \infty)$

9. List all of the critical points of the function $f(x) = \dfrac{e^x}{x^2-1}$. 3.1 #33

FREE A) $x = 0, \pm 3$ B) $x = \pm\sqrt{3}$ C) $x = -1, 3, \pm\sqrt{3}$ D) $x = -1, 3$

10. Calculate $\lim\limits_{h\to 0} \dfrac{(3+h)^2 - 3^2}{h}$. 1.5 #64

A) 6 B) 3 C) 9 D) 0

11. For which function does the second derivative test FAIL at $x = 0$? *3.3 #10*

A) $f(x) = x^4$ B) $f(x) = 3x + 1$ C) $f(x) = x^2$ D) $f(x) = x^2 + x^3$

12. Calculate $\lim_{x \to 1} \frac{\sin(\ln x)}{x - 1}$

A) ∞ B) 0 C) $\frac{\pi}{2}$ D) 1

13. If f is differentiable and increasing on $[0, 5]$, then which is the largest? *2.1 #15*

A) $\frac{f(3) - f(1)}{2}$ C) Not enough information is given.

B) $\lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$ D) $\lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$

14. If $\cos\theta = \frac{1}{5}$ and $\sin\theta < 0$, then what is $\sin\theta$? *0.4 #51*

A) $\frac{\sqrt{24}}{5}$ B) $\frac{1}{5}$ C) $\frac{2}{\sqrt{5}}$ D) $\frac{\sqrt{5}}{2}$

15. Suppose a rectangular ostrich pen is to be built with 540 feet of fencing material, and alongside a river (so that only three sides of fencing are required). Which of the following functions could be optimized in order to find the largest possible area of such a pen? *3.4 #36*

A) $f(x) = 2x + (540 - 2x)$ C) $f(x) = x(540 - 2x)$

B) $f(x) = x(270 - x)$ D) $f(x) = 2x + (\frac{540}{x})$

16. Which differentiation rule below did we prove WITHOUT using the definition of derivative? *2.3 reading*

A) $\frac{d}{dx}(\sin x) = \cos x$ C) $\frac{d}{dx}(\ln x) = \frac{1}{x}$

B) $\frac{d}{dx}(f(x) + g(x)) = \frac{df}{dx} + \frac{dg}{dx}$ D) $\frac{d}{dx}(x^k) = kx^{k-1}$

17. Calculate $\lim_{x \to \infty} (x - \sqrt{x})$. *1.6 #39*

A) 0 B) $\sqrt{x}$ C) ∞ D) $-\infty$