

List of Techniques and Subcases for Integration

Not guaranteed to be complete!! Provided for your convenience only, not as a replacement for your brain.

- Memorize how to integrate:
 - Constants k , power functions x^k , and exponential functions e^x , e^{kx} , b^x .
 - Integrands that are the derivatives of the six trig functions
 - Integrands that are the derivative of the inverse trig functions $\sin^{-1} x$, $\tan^{-1} x$, or $\sec^{-1} x$
 - Integrands that are the derivatives of hyperbolic functions (may look up signs)
 - Integrands that are the derivatives of $\sinh^{-1} x$, $\cosh^{-1} x$, or $\tanh^{-1} x$ (may look these up)
- Algebra:
 - Splitting a fraction into two or more fractions
 - Simplifying exponents and logarithms using algebraic rules
 - Multiplying out the integrand
 - Applying trig identities (or other identities) to simplify
 - Completing the square
- Tricks:
 - Adding and subtracting 1 (or some other value) somewhere in the integrand
 - Multiplying and dividing by the same expression (e.g. $\frac{x}{x}$ or $\frac{e^x}{e^x}$ or $\frac{\sec x + \tan x}{\sec x + \tan x}$)
- u -substitution:
 - u is “inside” another function and you can see u' in the integrand as a multiplicative factor
 - u isn't “inside” anything but you can see u' in the integrand as a multiplicative factor
 - u -substitution with back-substitution for x
 - Easy u -substitutions where u is linear and you can do it in your head
 - u -substitutions where u and u' are not obvious just by glancing at the integrand
- Inverse trig functions:
 - Integrals where inverse trig functions appear in the integrand
 - The integrand is the derivative of an inverse trig function after a u -substitution
 - The integrand is the derivative of an inverse trig function after factoring out an additive constant
- Hyperbolic functions (and inverse hyperbolic functions):
 - Integrals where hyperbolic functions (or their inverses) appear in the integrand
 - The integrand is the derivative of an inverse hyperbolic function (after some work)

- Integral tables and reduction formulae:
 - Examples we don't know how to integrate by any method *except* by a table. (Are there any?)
 - Examples where a reduction formula can be used

- Integration by parts:
 - u is same or better after differentiating, dv is same or better after integrating
 - Choosing u as the whole integrand
 - Choosing dv to be the “largest part” of the integrand that you can integrate
 - Examples where integration by parts has to be done more than once
 - Examples where you get the original integral back and have to solve for it

- Powers and products of trig functions:
 - Even powers of $\sin x$ or $\cos x$
 - Odd powers of $\sin x$ or $\cos x$
 - Integral of $\sin^m x \cos^n x$ where m or n is odd
 - Integral of $\sin^m x \cos^n x$ where m and n are both even
 - Integrals of $\sin(mx) \cos(nx)$, $\sin(mx) \sin(nx)$, and $\cos(mx) \cos(nx)$
 - Powers of $\tan x$ or $\cot x$
 - Even powers of $\sec x$ or $\csc x$
 - Odd powers of $\sec x$ or $\csc x$
 - Integral of $\tan^m x \sec^n x$ or $\cot^m x \csc^n x$ where m is odd
 - Integral of $\tan^m x \sec^n x$ or $\cot^m x \csc^n x$ where n is even
 - Integral of $\tan^m x \sec^n x$ or $\cot^m x \csc^n x$ where m is even and n is odd

- Trigonometric substitution:
 - Examples where $x = \sin u$, $x = \tan u$, or $x = \sec u$
 - Examples where $x = a \sin u$, $x = a \tan u$, or $x = a \sec u$
 - Examples where we set some function of x to be $a \sin u$, $a \tan u$, or $a \sec u$

- Partial fractions:
 - Distinct linear factors in the denominator
 - Repeated linear factors in the denominator
 - Distinct quadratic factors in the denominator
 - Repeated quadratic factors in the denominator

- Special cases:
 - Integrals that don't really fit any of the cases above
 - Integrals that require a trick or some special insight
 - Especially difficult or long integrals