Chapter 10 Outline

Math 236 Spring 2001

10.1 The Least Upper Bound Axiom

- **Upper and lower bounds:** Know the basic mathematical definitions of upper bound, lower bound, *lub*, *glb*. Be able to find the upper bounds of a given set, both when the set is given in interval notation and when the set is described with set notation. Be able to *prove* these bounds using the definitions.
- **Properties of *lub* and *glb*:** Know and be able to use Axiom 10.1.2 and Theorem 10.1.4. Be able to use *and prove* Theorems 10.1.3 and 10.1.5.

10.2 Sequences of Real Numbers

- **Sequences:** What is a sequence? Explain how you can think of a sequence as a function. Be able to go from a list of numbers to a general term and back.
- **Monotonicity:** What are the four types of monotonicity? Give definitions and examples for each of these types. Be able to tell if a sequence is monotonic, and if so what kind of monotonic. Also be able to *prove* this in simple cases.
- **Recursively defined sequences:** Don't forget that some sequences can be defined recursively; see the homework problems in this and other sections. You should be able to use the recursive definition to find a formula for the general term of the sequence (in terms of n only).

10.3 Limit of a Sequence

- **Convergence and divergence:** Know the limit definition of convergence. What does it mean for a sequence to diverge? Give examples. Give an example where the limit of a function defined on the positive integers (*i.e.* a sequence) converges but the limit of the same function defined on the real numbers diverges. You can do this with a picture if you like. Be able to determine if given sequence converges or diverges.
- **Bounded and unbounded sequences:** Know that a convergent sequence *must* be bounded (but not all bounded sequences are convergent!). Equivalently, every unbounded sequence is divergent.
- **Converging to *lub* or *glb*:** Look at Theorem 10.3.6. Be able to use this theorem to show that a given series is convergent to its *lub* or its *glb*. Draw a picture illustrating why this theorem is true. Does *every* convergent sequence converge to the *lub* or *glb* of the sequence? (The answer is no; why?)
- **The Pinching Theorem:** Be able to show that a sequence converges using the Pinching Theorem (10.3.9). Give an example of a sequence where you *need* the Pinching Theorem to show convergence.

10.4 Some Important Limits

- **Knowing the limits:** Memorize the seven special limits in this section. They will be used often. Be able to solve a limit by using algebra to turn it into one of these special limits.

- **Proving the limits:** You should know how to prove the special limits that involve the "logarithm trick" in the proof (10.4.1, 10.4.6, 10.4.7). You should also be able to apply that logarithm trick to a given limit that you are trying to compute.

10.5 The Indeterminate Form $\frac{0}{0}$

- **L'Hôpital's Rule:** Be able to give a mathematically precise statement of L'Hôpital's Rule in the $\frac{0}{0}$ case. Also be able to use this rule (only when it applies!) to solve limits.
- **Computational hints:** Only use L'Hôpital's Rule when it applies! Don't forget that if you can do algebra instead to solve the limit, you usually should (L'Hôpital's rule doesn't always work out in those cases). You can use L'Hôpital's Rule more than once on the same limit. Sometimes moving things around algebraically first makes L'Hôpital's Rule work out better.

10.6 The Indeterminate Form $\frac{\infty}{\infty}$; Other Indeterminate Forms

- **L'Hôpital's Rule (again):** Give a mathematical statement of L'Hôpital's Rule in the $\frac{\infty}{\infty}$ case. Be able to use this rule to solve limits (when it applies!).
- **The indeterminate form $0 \cdot \infty$:** Change this form into a L'Hôpital's Rule form by moving things from the numerator to the denominator (or vice-versa). Make sure after you move things around that what you have is equal to your original expression! Give different examples showing that a $0 \cdot \infty$ limit could be equal to 0, 1, ∞, or something else.
- **The indeterminate form $\infty - \infty$:** Combine terms to get a fraction (then algebra or L'Hôpital's Rule). Alternatively, multiply by a conjugate of sorts to get a fraction. Give different examples of $\infty - \infty$ limits that equal 0, 1, and ∞.
- **The indeterminate forms 0^0, 1^∞, and ∞^0:** Use the logarithm trick to solve these limits. Make sure you make clear that you are solving a *different* limit and then using the answer to that limit to answer the original limit.
- **Things that look like, but *are not* indeterminate forms:** The forms $\frac{0}{\infty}$, $\frac{\infty}{0}$, 0^∞, and $\infty + \infty$ look like they might be indeterminate forms, but they are *not*. What are these limits equal to and why? Can you think of any other forms that look like, but *are not* indeterminate?

10.7 Improper Integrals

- **Proper integrals:** What is a *proper* integral? What makes an integral *improper*? Give examples.
- **Integrals over infinite intervals:** Give a definition of the improper integral from $x = 1$ to $x = \infty$ of a function $f(x)$ in terms of a limit and a proper integral. Be able to do this to calculate such improper integrals. When taking limits from $-\infty$ to ∞ you have to split the integral somewhere in the middle. In that case if either piece diverges we say the entire thing diverges.

- **Integrals over discontinuities:** When integrating over a discontinuity (like a vertical asymptote), you must split the integral *at* the discontinuity. To solve an improper integral with a discontinuity at one end *only* we use a limit. Write down such an improper integral using a limit and a proper integral. Again, when splitting integrals, the whole integral diverges if any piece does.

- **Comparison test for integrals:** Be able to state and use the comparison test for integrals (see 10.7.2). Draw pictures illustrating why the comparison test for integrals works. Often we choose one of the integrals in (10.7.1) to compare with; you should memorize this type of integral and when it converges.

- **Tips for improper integrals:** Remember that you can only solve an improper integral that is bad on *one side*. If it's bad on two sides or in the middle then you have to split it up. Work on one integral at a time because if it diverges then you are done. Try to choose the one you think is most likely to diverge. Don't forget to check for discontinuities whenever you integrate a function!