TEST I

Math 236
February 8, 2001

Name: _____

By writing my name I swear by the honor code.

Read all of the following information before starting the exam:

- Circle or otherwise indicate your final answers.
- Show all work, clearly and in order. I will take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible. For most problems, work done by calculator will not receive any points (although you may use your calculator to check your answers).
- When you do use your calculator, sketch all relevant graphs and explain how you use them.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 9 problems and is worth 100 points, plus some extra credit at the end and for drawing a fish on the scrap page. Make sure that you have all of the pages!
- Good luck!

1. (*18 points*) Give definitions for the following expressions or properties. Your answers should be short mathematical expressions or statements, *not* complete sentences!!

(**a**) A function $f(x)$ is said to be *one-to-one* if:

(**b**) Given a one-to-one function $f(x)$, a function $g(x)$ is said to be the *inverse* of $f(x)$ if:

(**c**) For any positive x, the definition of $\ln x$ is:

(**d**) The definition of the number e is:

(**e**) Given any x, the the definition of e^x is:

(**f**) Given any positive x and any r, the definition of x^r is:

(**g**) Given any $b > 0$ with $b \neq 1$, the definition of $\log_b x$ is:

(**h**) The definition of the function $\sin^{-1} x$ is:

(**i**) The definition of the function $\cosh x$ is:

2. (*7 points*) Find the exact value of $\sin(\sec^{-1}(-\frac{1}{2}))$. Show your work. Your work should involve a sketch of the unit circle and a triangle.

3. (*7 points*) Find by hand all values of x for which $(2 - \ln x) \ln x = 0$.

4. (*7 points*) Prove that $\sinh t$ is an even function.

5. (*9 points*) Prove that $\frac{d}{dx}(e^x) = e^x$ using implicit differentiation and the fact that $\frac{d}{dx}(\ln x) = \frac{1}{x}$.

6. (*12 points*) Find the derivatives of the following functions. Show any work.

(**a**) $f(x) = x^2 \sec^{-1}(\frac{1}{x})$

(**b**) $f(x) = (\ln x)^x$

(**c**) $f(x) = 4^{(3x^2)}$

7. (*20 points*) Compute the following integrals. Show all work clearly (in particular, do not use memorized formulas for the first and last integrals).

(**a**) $\displaystyle\int \tan x \, dx$

(**b**) $\displaystyle\int_0^1 x 10^{1+x^2} \, dx$

(**c**) $\displaystyle\int \frac{\sinh x}{e^x} \, dx$

(**d**) $\displaystyle\int \frac{\sec^2 x}{\sqrt{9 - \tan^2 x}} \, dx$

8. (*8 points*) At what rate r of continuous compounding does a sum of money triple in 20 years?

9. (*12 points*) Find the domains of the following functions. Justify your answers.

(**a**) $\csc^{-1} x$

(**b**) $\dfrac{1}{\ln((x+1)^3)}$

(**c**) $\dfrac{1}{\cosh x}$

(**d**) The inverse of $f(x) = \frac{1}{x^3+1}$.

Survey Question (2 Extra Credit Points):

How did you do? Were you prepared for the kinds of questions that were on this test?

SCRAP WORK