

TEST II

Math 236
March 15, 2001

Name: _____

By writing my name I swear by the honor code.

Read all of the following information before starting the exam:

- Circle or otherwise indicate your final answers.
- Show all work, clearly and in order. I will take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible. For most problems, work done by calculator will not receive any points (although you may use your calculator to check your answers).
- When you do use your calculator, sketch all relevant graphs and explain how you use them.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements but will add a couple of points for circling the word rambling in this sentence.
- This test has 7 problems and is worth 100 points, plus some extra credit at the end. Make sure that you have all of the pages!
- Good luck!

1. (24 points) Please solve the following integrals. Show your work clearly and in order and circle your final answers.

a. (8 pts) $\int \sin^2 x \cos^5 x \, dx$

b. (8 pts) $\int \frac{2x-1}{\sqrt{x+3}} \, dx$

c. (8 pts) $\int \sin^{-1} x \, dx$

2. (6 points) Set up an Initial Value Problem whose solution will be a function that you could use to solve the following word problem. Be sure that you define any variables or functions that you name.

A flu virus is spreading rapidly through a small town with a population of 25,000. The disease is spreading at a rate proportional to the product of the number of people who have it and the number who don't. Suppose that 100 people had the disease initially, and 400 had it after 10 days. How long will it take for half the population of the town to have the flu?

DO NOT SOLVE THE IVP OR TRY TO SOLVE THE WORD PROBLEM.

3. (12 points) For each differential equation listed below, list **ALL** of the descriptions **A**, **B**, or **C** that apply. If none of **A**, **B**, or **C** apply, write "NONE" in the blank.

A. This is a first-order linear differential equation.

B. This is a separable differential equation.

C. The solutions to this differential equation are exponential.

D. This differential equation is just an antidifferentiation problem.

(a) _____ $\frac{dr}{ds} = 4r$

(b) _____ $\frac{dr}{ds} = 4r - 5$

(c) _____ $\frac{dr}{ds} = 4s - 5$

(d) _____ $\frac{dW}{dt} = \frac{2W + t^2}{t}$

(e) _____ $\frac{dy}{dx} = \sin(xy)$

(f) _____ $\frac{dP}{dt} = P - 3P^2$

4. (16 points) Please solve the following integrals. Show your work clearly and in order and circle your final answers.

a. (8 pts) $\int \frac{1}{x^2\sqrt{x^2-4}} dx$

b. (8 pts) $\int \frac{x^2}{(x-1)^2(x+1)} dx$

5. (*18 points*) Solve the differential equation $\frac{dy}{dx} = 2 - 3y$ two different ways. Circle your final answers to (a) and (b).

a. (*7 pts*) Use the method for first-order linear differential equations.

b. (*7 pts*) Use the method for separable differential equations.

c. (*4 pts*) Use algebra to show that your two answers above are the same.

6. (12 points) Match each integral with **ONE** integration technique that would be effective for that integral. Use each choice from **A–F** exactly once.

(a) _____ $\int \frac{x^3}{x^2 + 1} dx$

(b) _____ $\int \frac{1}{x^2 + 1} dx$

(c) _____ $\int \frac{x}{x^2 - x - 2} dx$

(d) _____ $\int x \sin x^2 dx$

(e) _____ $\int x^2 \sin x^2 dx$

(f) _____ $\int x^3 \sin x^2 dx$

A. integration by parts

B. u -substitution

C. partial fractions

D. trigonometric substitution

E. inverse trig functions

F. can't solve this integral

7. (12 points) Fill in the blanks with short answers to the following questions.

(a) The integration by parts formula is derived from the _____ rule.

(b) $\int (e^{x^2} y' + 2xe^{x^2} y) dx = \text{_____} + C.$

(c) In terms of exponential functions, $\operatorname{sech} x^2 = \text{_____}.$

(d) If u is a function of x then $du = \text{_____}.$

(e) $\int y'(u(x)) u'(x) dx = \text{_____} + C.$

Survey Question (2 Extra Credit Points):

Do you think this test was hard? Do you think you did well?

SCRAP WORK