TEST III

Math 236
April 19, 2001

Name: ______________________________

By writing my name I swear by the honor code.

Read all of the following information before starting the exam:

- Circle or otherwise indicate your final answers.
- Show all work, clearly and in order. I will take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible. For most problems, work done by calculator will <u>not</u> receive any points (although you may use your calculator to check your answers).
- When you do use your calculator, sketch all relevant graphs and explain how you use them.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 9 problems and is worth 100 points, plus some extra credit at the end and for drawing a tree on the scrap page. Make sure that you have all of the pages!
- Good luck!

1. (*10 points*) Find the sum of the series $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2+3k+2}$. Show your work clearly and in order.

2. (*10 points*) Fill in the boxes with the appropriate mathemtatical statements.

(**a**) A sequence $\{a_n\}$ is *nondecreasing* if $\boxed{}$.

(**b**) If L is the least upper bound of a set S and ϵ is any positive number, then there exists at least one x in S such that $\boxed{}$.

(**c**) An definite integral $\displaystyle\int_a^b f(x)\,dx$ is a *proper* integral (*i.e.* not an improper integral) if $\boxed{}$ and $\boxed{}$.

(**d**) It is true that $\displaystyle\sum_{k=1}^{\infty} f(k)$ converges if and only if $\displaystyle\int_1^{\infty} f(x)\,dx$ converges, as long as f is $\boxed{}$, $\boxed{}$, and $\boxed{}$ on $[1, \infty)$.

(**e**) The Integral, Comparison, Limit Comparison, Root, and Ratio Tests all require that the terms of $\sum a_k$ be $\boxed{}$.

3. (*20 points*) Determine if the following series converge or diverge. Justify each of your answers.

a. (*5 pts*) $\sum \left(\frac{1}{k}\right)^2$

b. (*5 pts*) $\sum \frac{1}{2^k}$

c. (*10 pts*) $\sum \frac{3^{2k}}{(2k)!}$

4. (*10 points*) Let $\sum a_k$ be an infinite series with sequence of terms $\{a_k\}$ and sequence of partial sums $\{s_n\}$. Circle True (**T**) or False (**F**) for each of the statements below.

(**a**) **T** **F** If $\{s_n\}$ diverges, then $\sum a_k$ diverges.

(**b**) **T** **F** If $\{s_n\}$ converges to 4, then $\sum a_k$ converges to 4.

(**c**) **T** **F** If $\sum a_k$ diverges then $\{a_k\}$ diverges.

(**d**) **T** **F** If $\{a_k\}$ does not converge to 0, then $\{s_n\}$ diverges.

(**e**) **T** **F** If $\sum |a_k|$ diverges, then $\sum a_k$ diverges.

5. (*10 points*) Determine if the series $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^3}$ converges or diverges. Show all work.

6. (*14 points*) Let $\sum a_k$ be a series with nonnegative terms. Circle True (**T**) or False (**F**) for each of the statements below.

(**a**) **T** **F** If $\displaystyle\lim_{k\to\infty} (a_k)^{\frac{1}{k}} = 2$, then $\sum a_k$ diverges.

(**b**) **T** **F** If $\displaystyle\lim_{k\to\infty} a_k = 0$, then $\sum a_k$ converges.

(**c**) **T** **F** If $\displaystyle\lim_{k\to\infty} \frac{a_k}{1/k} = 1$, then $\sum a_k$ diverges.

(**d**) **T** **F** If $\displaystyle\lim_{k\to\infty} \frac{a_{k+1}}{a_k} = 0$, then $\sum a_k$ converges.

(**e**) **T** **F** If $\displaystyle\lim_{k\to\infty} (a_k)^{\frac{1}{k}} = 1$, then $\sum a_k$ converges.

(**f**) **T** **F** If $\displaystyle\lim_{k\to\infty} \frac{a_k}{(.5)^k} = 0$, then $\sum a_k$ converges.

(**g**) **T** **F** If $\displaystyle\lim_{k\to\infty} \frac{a_{k+1}}{a_k} = \frac{1}{2}$, then $\sum a_k$ converges.

7. (*10 points*) Is the sequence $\{(k^3+1)^{\frac{1}{\ln k}}\}$ convergent (assume $k \geq 2$)? If so, what does it converge to? If not, why not? Justify your answer.

8. (*8 points*) Suppose $\{a_k\}$ is a sequence of real numbers. Circle True (**T**) or False (**F**) for each of the statements below.

(**a**) **T** **F** If $\{a_k\}$ is bounded, then it is convergent.

(**b**) **T** **F** If $\{a_k\}$ is not bounded, then it is divergent.

(**c**) **T** **F** If $\{a_k\}$ is convergent and bounded above then it converges to its *lub*.

(**d**) **T** **F** If $\{a_k\}$ is bounded and nonincreasing then it converges to its *glb*.

9. (*8 points*) Suppose $\sum a_k$ and $\sum b_k$ are infinite series with the property that $0 \leq a_k \leq b_k$ for all k. Circle True (**T**) or False (**F**) for each of the following statements.

(**a**) **T** **F** If $\sum a_k$ converges, then $\sum b_k$ converges.

(**b**) **T** **F** If $\sum b_k$ diverges, then $\sum a_k$ converges.

(**c**) **T** **F** If $\sum a_k$ diverges, then $\sum b_k$ diverges.

(**d**) **T** **F** I hate True/False questions.

Survey Question (2 Extra Credit Points):

How did you do on this test? Which question was the hardest?

SCRAP WORK