List of Techniques and Subcases for Integration **236 Summer 2002**

Not guaranteed to be complete!! Provided for your convenience only, not as a replacement for your brain.

- Memorize how to integrate:
  - Constants k, power functions x^k, and exponential functions e^x, e^{kx}, b^x.
  - Integrands that are the derivatives of the six trig functions
  - Integrands that are the derivatives of the inverse trig functions $\sin^{-1} x$, $\tan^{-1} x$, or $\sec^{-1} x$
  - Integrands that are the derivatives of hyperbolic functions.
  - Integrals of $\tan x$, $\cot x$, $\sec x$, $\csc x$, and $\ln x$. (but also be able to do by hand)

- Algebra:
  - Splitting a fraction into two or more fractions
  - Simplifying exponents and logarithms using algebraic rules
  - Multiplying out the integrand
  - Applying trig identities (or other identities) to simplify
  - Completing the square

- Tricks:
  - Adding and subracting 1 (or some other value) somewhere in the integrand
  - Multiplying and dividing by the same expression (*e.g.* $\frac{x}{x}$ or $\frac{e^x}{e^x}$ or $\frac{\sec x+\tan x}{\sec x+\tan x}$)

- u-substitution:
  - u is "inside" another function and you can see u' in the integrand as a multiplicative factor
  - u isn't "inside" anything but you can see u' in the integrand as a multiplicative factor
  - u-substitution with back-substitution for x
  - Easy u-substitutions where u is linear and you can do it in your head
  - u-substitutions where u and u' are not obvious just by glancing at the integrand

- Inverse trig functions:
  - Integrals where inverse trig functions appear in the integrand
  - The integrand is the derivative of an inverse trig function after a u-substitution
  - The integrand is the derivative of an inverse trig function after factoring out an additive constant

- Hyperbolic functions (and inverse hyperbolic functions):
  - Integrals where hyperbolic functions (or their inverses) appear in the integrand
  - The integrand is the derivative of an inverse hyperbolic function (after some work)

- Integral tables and reduction formulae:
  - Examples we don't know how to integrate by any method *except* by a table. (Are there any?)
  - Examples where a reduction formula can be used

- Integration by parts:
  - u is same or better after differentiating, dv is same or better after integrating
  - Choosing u as the whole integrand
  - Choosing dv to be the "largest part" of the integrand that you can integrate
  - Examples where integration by parts has to be done more than once
  - Examples where you get the original integral back and have to solve for it

- Powers and products of trig functions:
  - Even powers of $\sin x$ or $\cos x$
  - Odd powers of $\sin x$ or $\cos x$
  - Integral of $\sin^m x \cos^n x$ where m or n is odd
  - Integral of $\sin^m x \cos^n x$ where m and n are both even
  - Powers of $\tan x$ or $\cot x$
  - Even powers of $\sec x$ or $\csc x$
  - Odd powers of $\sec x$ or $\csc x$
  - Integral of $\tan^m x \sec^n x$ or $\cot^m x \csc^n x$ where m is odd
  - Integral of $\tan^m x \sec^n x$ or $\cot^m x \csc^n x$ where n is even
  - Integral of $\tan^m x \sec^n x$ or $\cot^m x \csc^n x$ where m is even and n is odd

- Trigonometric substitution:
  - Examples where $x = \sin u$, $x = \tan u$, or $x = \sec u$
  - Examples where $x = a \sin u$, $x = a \tan u$, or $x = a \sec u$
  - Examples where we set some function of x to be $a \sin u$, $a \tan u$, or $a \sec u$

- Partial fractions:
  - Distinct linear factors in the denominator
  - Repeated linear factors in the denominator
  - Distinct quadratic factors in the denominator
  - Repeated quadratic factors in the denominator

- Special cases:
  - Integrals that don't really fit any of the cases above
  - Integrals that require a trick or some special insight
  - Especially difficult or long integrals