

8.1–8.5 Integration Techniques

In addition to being able to integrate quickly and accurately using all the techniques in 8.1–8.5, you should know how to explain how integration by substitution and integration by parts are really the chain and product rules “backwards”, respectively. You should know by heart the three pythagorean trig identities and the “double angle” identities for $\sin^2 x$ and $\cos^2 x$. More complicated trig identities will be provided for you if needed. Be sure that you have memorized all of the basic integrals listed on the integration handout. Note that Section 8.6 has been dropped from the syllabus and therefore will not be covered on the exam.

8.7 Numerical Integration

- **Various approximations of definite integrals:** Be able to approximate definite integrals using the left-hand sum rule, the right-hand sum rule, the midpoint rule, and the trapezoid rule. Notation is not necessarily needed to do these calculations.
- **Notation for the approximations:** Know the notation involved in approximations for the definite integral (for example, x_i , Δx , $f(x_i^*)$, and so on). Be able to write and explain the general formulae for the approximations listed above. You do *not* have to be able to write these approximations using sigma notation.
- **Error:** Be able to bound the error involved in the approximations above using the various error formulae. The error-bounding formulae will be provided for you.

8.8-8.9 Differential Equations

- **Basic differential equations:** What is a differential equation? What is an Initial Value Problem? What is the solution to a differential equation? To an Initial Value Problem? What kind of differential equations can be solved by anti-differentiation? (In other words, which differential equations are just integration problems?) What is the general solution to $y' = ky$ and why? Be able to solve such simple differential equations and be able to check if a given function is the solution to a differential equation or Initial Value Problem.
- **First-order linear differential equations:** What is a first-order linear differential equation? Be able to write first-order linear equations in the form $y' + p(x)y = q(x)$, and solve them using the appropriate integration factor. Be sure that you can explain why each step in this technique is legal.
- **Separable differential equations:** What is a separable differential equation? Be able to write separable differential equations in “separated” form, and solve them by “integrating both sides”. Be able to explain this process in terms of the chain rule (why does the y stuff have to be *multiplied* by $\frac{dy}{dx}$?).
- **Constructing Initial Value Problems:** Given a word problem, be able to construct a differential equation and initial condition whose solution will be a function asked for in the problem. Then be able to solve this Initial Value Problem and use the solution to answer the word problem.