TEST I

Math 236
May 22, 2002

Name: _____
By writing my name I swear by the honor code.

Read all of the following information before starting the exam:

- Show all work, clearly and in order. I will take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Make sure that you follow the directions in each problem and that your answer matches what is asked for.
- Justify your answers algebraically whenever possible. For most problems, work done by calculator will not receive any points (although you may use your calculator to check your answers).
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 6 problems and is worth 100 points. Make sure that you have all of the pages!
- Good luck!

1. (*16 points*) Fill in the blanks.

a. (*3 pts*) The ***definition*** of the natural logarithm function is $\ln x :=$ _____ .

b. (*3 pts*) The ***definition*** of the base b logarithm function is $\log_b x :=$ _____ .

c. (*3 pts*) The ***definition*** of the number e is _____ .

d. (*3 pts*) The function $\sin^{-1} x$ has domain _____ and range _____ .

e. (*3 pts*) Write the function $\tanh x$ in terms of e^x and e^{-x}: $\tanh x =$ _____ .

f. (*1 pt*) My favorite food in the whole world is _____ .

2. (*16 points*) Show all work for each problem below (no calculators!); circle your final answers.

a. (*4 pts*) Solve $\log_x 2 = \log_3 x$.

b. (*4 pts*) Find the domain of $\dfrac{1}{\ln(\ln x)}$.

c. (*4 pts*) Find the exact value of $\cosh(\ln 2)$.

d. (*4 pts*) Find the exact value of $\sec^{-1}(-2)$.

3. (*24 points*) Differentiate the following functions. Show all work and circle your final answers.

a. (*4 pts*) $f(x) = e^{3\sin x}$

b. (*4 pts*) $f(x) = \sin^{-1}\sqrt{x}$

c. (*4 pts*) $f(x) = \log_3(2^x)$

d. (*4 pts*) $f(x) = \operatorname{sech}(\ln x)$

e. (*4 pts*) $f(x) = \ln(\sinh x)$

f. (*4 pts*) $f(x) = x^{\cos x}$

4. (*8 points*) Dr. Drosophila has noticed that the number of fruit flies in his lab triples every 4 hours. What is the doubling time for Dr. Drosophila's fruit fly population?

5. (*12 points*) In this problem you will prove that $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$, in two steps:

a. (*8 pts*) Use the fact that $\tan(\tan^{-1} x) = x$ to prove that:

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{\sec^2(\tan^{-1} x)}.$$

b. (*4 pts*) Use a triangle (or triangles) to show that for all x:

$$\sec^2(\tan^{-1} x) = 1 + x^2.$$

6. (*24 points*) Solve the following integrals. Show all work and circle your final answers.

a. (*4 pts*) $\displaystyle\int \frac{3}{2^x}\,dx$

b. (*4 pts*) $\displaystyle\int \frac{1}{9+4x^2}\,dx$

c. (*4 pts*) $\displaystyle\int \frac{\sinh\sqrt{x}}{\sqrt{x}}\,dx$

d. (*4 pts*) $\displaystyle\int \frac{1}{x(\ln x)^2}\,dx$

e. (*4 pts*) $\displaystyle\int x\,\operatorname{sech}^2(x^2)\,dx$

f. (*4 pts*) $\displaystyle\int \frac{e^x}{3-2e^x}\,dx$

Survey Questions: (2 extra credit points)

Name a question or topic that could have been on this test, but wasn't.

How do you think you did?

SPACE FOR SCRAP WORK