TEST II

Math 236
June 4, 2002

Name: ______________________________
By writing my name I swear by the honor code.

Read all of the following information before starting the exam:

- Show all work, clearly and in order. I will take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Make sure that you follow the directions in each problem and that your answer matches what is asked for.
- Justify your answers algebraically whenever possible. For most problems, work done by calculator will not receive any points (although you may use your calculator to check your answers).
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 7 problems and is worth 100 points. Make sure that you have all of the pages!
- Good luck!

1. (*16 points*) Fill in the blanks.

a. (*4 pts*) If $\sqrt{1}(2)+\sqrt{3}(2)+\sqrt{5}(2)$ is a Midpoint Sum approximation for $\int_a^b \sqrt{x}\,dx$,

then $a =$ _______ and $b =$ _______ .

b. (*4 pts*) Consider the integral $\int \cos^2(3x)\,dx$.

To solve this integral you need the trignometric identity: $\cos^2(3x) =$ ______________ .

c. (*4 pts*) Write $\sec^2(\sin^{-1}(\frac{x-1}{3}))$ as an algebraic function: __________________ .

d. (*4 pts*) Use polynomial long division to rewrite $\dfrac{x^3-2x^2+1}{x^2+3} =$ _____________________ .

2. (*16 points*) Fill in the blanks.

a. (*4 pts*) Consider the differential equation $\dfrac{xy+3y'}{x^2+1} = x$.

The integrating factor for this differential equation is: ______________ .

b. (*4 pts*) Consider the differential equation $\dfrac{dy}{dx} = 3+2y$.

Write this differential equation in "separated" form: _____________________ .

c. (*4 pts*) Consider the differential equation $\dfrac{dy}{dx} + y^2 = x$.

Can this differential equation be written in first-order linear form? _______ .

Is this differential equation separable? _______ .

d. (*4 pts*) A drug is fed intervenously into a patient's bloodstream at the constant rate r. Simultaneously, the drug leaves the patient's body at a rate proportional to the amount of drug present. Write down the differential equation that describes the amount $Q(t)$ of drug in the patient's bloodstream at time t:

$$\frac{dQ}{dt} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ .$$

3. (*6 points*) Show that $\int \sec x \, dx = \ln|\sec x + \tan x|$.

4. (*24 points*) Consider the following nine integrals.

A. $\displaystyle\int \frac{2}{x^2-1}\,dx$	**D.** $\displaystyle\int e^{x^2+1}\,dx$	**G.** $\displaystyle\int \sec^3 x\,dx$
B. $\displaystyle\int \frac{e^x+1}{\sqrt{e^x}}\,dx$	**E.** $\displaystyle\int x^3\cos(x^2)\,dx$	**H.** $\displaystyle\int \frac{1}{\sqrt{x^2-6x-7}}\,dx$
C. $\displaystyle\int \sin^5 x\cos^2 x\,dx$	**F.** $\displaystyle\int \frac{\ln x}{x^2}\,dx$	**I.** $\displaystyle\int \frac{\sinh x}{\cosh^2 x}\,dx$

For each integration technique below, choose ***one*** of the integrals above that can be solved using that integration technique. Then fill in the blanks to describe how the technique will work in that particular example. (Notice that you will have to identify *three* different integrals that can be done using integration by parts; also, one of the integrals will not be matched with any integration technique.)

DO NOT SOLVE ANY OF THE INTEGRALS!!

a. (*3 pts*) ______ Algebra, by rewriting the integrand as: ____________________ .

b. (*3 pts*) ______ Integration by substitution: $u =$ ____________ .

c. (*3 pts*) ______ Integration by parts: $u =$ ____________ , $dv =$ ____________ .

d. (*3 pts*) ______ Integration by parts: $u =$ ____________ , $dv =$ ____________ .

e. (*3 pts*) ______ Integration by parts: $u =$ ____________ , $dv =$ ____________ .

f. (*3 pts*) ______ Trigonometric identities, with the eventual substitution $u =$ ____________ .

g. (*3 pts*) ______ Trigonometric substitution, with: ____________________ .

h. (*3 pts*) ______ Partial fractions, by rewriting the integrand as: ____________________ .

5. (*24 points*) Solve each of the following integrals. Show your work and circle your final answers.

a. (*8 pts*) $\displaystyle\int \frac{x+1}{x^4+x^2}\,dx$

b. (*8 pts*) $\displaystyle\int \tan^3 x\,dx$

c. (*8 pts*) $\displaystyle\int \frac{1}{x\sqrt{x^2-4}}\,dx$

6. (*7 points*) Find the smallest integer n that will guarantee that an approximation of $\int_2^7 \ln x \, dx$ with n trapezoids will be accurate to within 0.01. (Use the error bound formula below.)

$$|E_n^T| \leq \frac{(b-a)^3}{12n^2} M, \quad \text{where } M \geq |f''(x)| \text{ for all } x \in [a,b].$$

7. (*7 points*) Use the product rule to prove the integration by parts formula. The first and last lines of the proof are given; you fill in the rest.

Proof:
By the product rule for differentiation, if $u(x)$ and $v(x)$ are differentiable functions, then:

$$\frac{d}{dx}(u(x)v(x)) = u'(x)v(x) + u(x)v'(x).$$

Therefore we have: $$\int u \, dv = uv - \int v \, du.$$ ■

Survey Questions: (2 extra credit points)

Name a question or topic that could have been on this test, but wasn't.

How do you think you did?

SPACE FOR SCRAP WORK