TEST III

Math 236
June 13, 2002

Name: _____

By writing my name I swear by the honor code.

Read all of the following information before starting the exam:

- Show all work, clearly and in order. I will take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Make sure that you follow the directions in each problem and that your answer matches what is asked for.
- Justify your answers algebraically whenever possible. For most problems, work done by calculator will <u>not</u> receive any points (although you may use your calculator to check your answers).
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 8 problems and is worth 100 points. Make sure that you have all of the pages!
- Good luck!

1. (*10 points*) Circle either True (T) or False (F) for each statement below.

a. (*2 pts*) **T** **F** Every convergent sequence is bounded.

b. (*2 pts*) **T** **F** Every monotonic sequence is convergent.

c. (*2 pts*) **T** **F** A sequence is said to be *nonincreasing* if it is not increasing.

d. (*2 pts*) **T** **F** If a sequence $\{a_k\}$ converges to 0, then the series $\sum a_k$ converges.

e. (*2 pts*) **T** **F** If $\{s_n\}$ is the sequence of partial sums for a series $\sum_{k=0}^{\infty} a_k$, and if $\lim_{n\to\infty} s_n = 2$, then $\sum_{k=0}^{\infty} a_k = 2$.

2. (*24 points*) Fill in the blanks or give short answers to each of the following questions.

a. (*4 pts*) State in mathematical notation what it means for a number M to be the *least upper bound* of a set S.

b. (*4 pts*) Show that the sequence $\{\frac{1}{n^2}\}$ is decreasing by looking at the ratio $\frac{a_{n+1}}{a_n}$.

c. (*4 pts*) Find the general form a_n of nth term of the sequence $\{-\frac{1}{4}, \frac{2}{9}, -\frac{3}{16}, \frac{4}{25}, -\frac{5}{36}, \ldots\}$.

$a_n =$ _____

d. (*4 pts*) Find the least upper bound (if one exists) and greatest lower bound (if one exists) of the set $\{x \mid x^2 + x - 2 < 0\}$.

$lub =$ _____ $glb =$ _____

e. (*4 pts*) Find the least upper bound (if one exists) and greatest lower bound (if one exists) of the sequence $\{(.9)^n\}$.

$lub =$ _____ $glb =$ _____

f. (*4 pts*) Find the approximate numerical value of the fourth term s_4 of the sequence of partial sums $\{s_n\}$ for the series $\displaystyle\sum_{k=0}^{\infty} \frac{1}{3^k}$. (You may use a calculator.)

$s_4 =$ _____

3. (*6 points*) Find the exact value of the limit $\lim_{x\to\infty}(e^x+1)^{\frac{1}{x}}$. Show your work.

4. (*18 points*) Determine whether or not each of the following sequences converges or diverges. If a sequence converges, find its limit. Justify your answers.

a. (*6 pts*) $\left\{\dfrac{4^n}{3^n+100}\right\}$

b. (*6 pts*) $\left\{\sqrt{n+1}-\sqrt{n}\right\}$

c. (*6 pts*) $\left\{\dfrac{\ln(\sec n)}{n^2}\right\}$

5. (*6 points*) Use the comparison test to determine whether or not the improper integral below converges or diverges. Show and explain your work.

$$\int_1^\infty \frac{x}{\sqrt{1+x^5}}\,dx$$

6. (*12 points*) Determine whether each integral below converges or diverges. If an integral converges, find its value. Show your work.

(**a**) $$\int_1^4 \frac{1}{x-2}\,dx$$

(**b**) $$\int_e^\infty \frac{1}{x(\ln x)^2}\,dx$$

7. (*6 points*) Find the exact sum of the series $\sum_{k=2}^{\infty} \frac{3^{k-2}}{4^{2k+1}}$.

8. (*18 points*) Determine whether each series converges or diverges. Show your work and make clear what tests you are using.

a. (*6 pts*) $\sum_{k=1}^{\infty} \frac{2+\sin k}{k^2}$

b. (*6 pts*) $\sum_{k=0}^{\infty} \frac{2k+1}{k^3+1}$

c. (*6 pts*) $\sum_{k=1}^{\infty} \frac{k}{e^{k^2}}$

Survey Questions: (2 extra credit points)

Name a question or topic that could have been on this test, but wasn't.

How do you think you did?

SPACE FOR SCRAP WORK