

236 Quiz 2

January 25, 2011

Name: * KEY *

Name: _____

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Work in groups but do NOT split up problems or tasks. You must discuss each problem as a group and agree on a final answer. Hand in one quiz per group.

You may use your hand-written Notebooks but no other materials and no technology at all. Please keep your discussions quiet so as not to disturb or inform other groups.

1. Solve each of the following integrals. Do your scratch work elsewhere but write the final version of your work clearly here. Put a box around your final answer.

a) $\int \frac{x^2 + x + 1}{(x^2 + 1)^2} dx$

$\int \left(\frac{1}{x^2 + 1} + \frac{x}{(x^2 + 1)^2} \right) dx$
 $(u = x^2 + 1, du = 2x dx)$

$\int \frac{1}{x^2 + 1} dx + \frac{1}{2} \int u^{-2} du$

$\tan^{-1} x + \frac{1}{2}(-1)(x^2 + 1)^{-1} + C$

PF: $\frac{x^2 + x + 1}{(x^2 + 1)^2} = \frac{Ax + B}{x^2 + 1} + \frac{Cx + D}{(x^2 + 1)^2}$

$(Ax + B)(x^2 + 1) + (Cx + D) = x^2 + x + 1$

$(A)x^3 + (B)x^2 + (A+C)x + (B+D) = x^2 + x + 1$

$\begin{cases} A=0 \\ B=1 \\ A+C=1 \\ B+D=1 \end{cases} \Rightarrow \begin{cases} A=0 \\ B=1 \\ C=1 \\ D=1-B=0 \end{cases}$

7 pts

b) $\int \sec^3 x \tan^3 x dx$

$\int \sec^2 x \tan^2 x (\sec x \tan x) dx$

$\int \sec^2 x (\sec^2 x - 1) (\sec x \tan x) dx$

$(u = \sec x, du = \sec x \tan x dx)$

$\int u^2(u^2 - 1) du = \int (u^4 - u^2) du$

$\frac{1}{5} u^5 - \frac{1}{3} u^3 + C = \frac{1}{5} \sec^5 x - \frac{1}{3} \sec^3 x + C$

7 pts

c) $\int x^3 \sin x^2 dx$

$\int x^2 \cdot x \sin x^2 dx$

$(u = x^2 \rightarrow du = 2x dx, v = -\frac{1}{2} \cos x^2 \leftarrow dv = x \sin x^2 dx)$

$-\frac{1}{2} x^2 \cos x^2 + \int x \cos x^2 dx$

$(w = x^2, dw = 2x dx)$

$\int x \sin x^2 dx$
 $(w = x^2, dw = 2x dx)$
 $\frac{1}{2} \int \sin w dw$
 $-\frac{1}{2} \cos x^2 + C$

$-\frac{1}{2} x^2 \cos x^2 + \frac{1}{2} \int \cos w dw = -\frac{1}{2} x^2 \cos x^2 + \frac{1}{2} \sin x^2 + C$

6 pts