



236 Quiz 3

February 8, 2011

P.2

Name \_\_\_\_\_

\* Key \*

By printing my name I pledge to uphold the Honor Code.

Work individually. You may use your Notebooks but no loose papers, photocopies, books, calculators, cell phones, or other resources.

Choose two of the three to integrate. Put one calculation in each region below. Do scrap work on a different page but on this page be sure to show all final work and put a box around your final answer.

$$\int \frac{1}{e^x \sqrt{9 + e^{2x}}} dx \quad \int \sec^3 x dx \quad \int \frac{3x^2 + 6}{(x+1)(x^2 - x + 1)} dx$$

THIRD

~~FIRST~~ INTEGRAL:

$$\int \frac{3x^2 + 6}{(x+1)(x^2 - x + 1)} dx = \int \left( \frac{A}{x+1} + \frac{Bx+C}{x^2 - x + 1} \right) dx$$

(irred. since  $b^2 - 4ac = 1 - 4 < 0$ )

$$= \int \left( \frac{3}{x+1} + \frac{3}{x^2 - x + 1} \right) dx$$

$$= 3 \ln|x+1| + 3 \int \frac{1}{(x - 1/2)^2 + 3/4} dx$$

$$= 3 \ln|x+1| + 3 \cdot \frac{4}{3} \int \frac{1}{\left(\frac{2}{\sqrt{3}}(x - 1/2)\right)^2 + 1} dx$$

$$= \boxed{3 \ln|x+1| + 4 \cdot \frac{\sqrt{3}}{2} \tan^{-1}\left(\frac{2}{\sqrt{3}}(x - 1/2)\right) + C}$$

PARTIAL FRACTIONS

$$\begin{aligned} A(x^2 - x + 1) + (x+1)(Bx+C) &= 3x^2 + 6 \\ (A+B)x^2 + (-A+C)x + (A+C) &= 3x^2 + 6 \\ \begin{cases} A+B=3 \Rightarrow B=3-A \\ -A+C=0 \\ A+C=6 \Rightarrow C=6-A \end{cases} \\ \rightarrow -A + (3-A) + (6-A) &= 0 \\ 9 &= 3A \\ 3 &= A \\ \text{So } B &= 3-3=0 \text{ and } C=6-3=3 \end{aligned}$$

SECOND INTEGRAL:

COMPLETE SQUARE

$$\begin{aligned} x^2 - x + 1 \\ (x^2 - x + 1/4) + (1 - 1/4) \\ \uparrow \\ (1/2)^2 \\ (x - 1/2)^2 + 3/4 \end{aligned}$$