(sums to 21)

236 Quiz 6

March 22, 2011 **Name** * key *

By printing my name I pledge to uphold the Honor Code.

Work on your own with only your notebook.

1. Explain what you would have to show to prove that the series

$$\sum_{k=1}^{\infty} \frac{k}{3^k}$$

converges, using each of the three tests listed below. You do NOT actually have to perform the tests, just describe what needs to be done.

7 pts each

a) Integral Test

▷ would have to solve the improper integral: $\int_1^\infty \frac{x}{3^x}\,dx = \int_1^\infty x\left(\frac{1}{3}\right)^x dx$

▷ using the integration technique: parts with $u=x$, $dv=\left(\frac{1}{3}\right)^x dx$

▷ and then make the conclusion that: integral converges, so series converges

b) Comparison Test

▷ could compare to the series: (one option) $\sum_{k=1}^{\infty}\left(\frac{2}{3}\right)^k$

▷ because of this inequality: $\left(\frac{2}{3}\right)^k = \frac{2^k}{3^k} \geq \frac{k}{3^k}$

▷ and then make the conclusion that: geom. $\sum\left(\frac{2}{3}\right)^k$ conv., so our series does too

c) Limit Comparison Test

▷ could compare to the series: $\sum_{k=1}^{\infty}\left(\frac{2}{3}\right)^k$ (one option)

▷ and would have to calculate this limit: $\lim_{k\to\infty} \frac{k/3^k}{2^k/3^k}$

▷ and then make the conclusion that: limit is a positive number (finite) and geom. $\sum\left(\frac{2}{3}\right)^k$ converges, so our series does too.