236 Quiz 7 Name:

March 29, 2011 Name: * key *

Name:

Work in groups but do NOT split up problems or tasks. You must discuss each problem as a group and agree on a final answer. Hand in one quiz per group.

You may use your hand-written Notebooks but no other materials and no technology at all. Please keep your discussions quiet so as not to disturb or inform other groups.

7 pts each

1. Determine the convergence or divergence of each of the following series, using the methods listed. Show your work carefully and make good arguments to support your answers.

a) $\displaystyle\sum_{k=1}^{\infty} \frac{k^3}{2^{\sqrt{k^3}}}$ ← use the Root Test; carefully justify algebra/limit calculations.

$$\lim_{k\to\infty} \left(\frac{k^3}{2^{\sqrt{k^3}}}\right)^{1/k} = \lim_{k\to\infty} \frac{k^{3/k}}{2^{k^{3/2}/k}} = \lim_{k\to\infty} \frac{k^{3/k}}{2^{\sqrt{k}}} \quad \frac{\to (k^{1/k})^3 \to 1}{\to 2^{\infty} \to \infty} \to 0.$$

since $0<1$ the Root Test says that $\displaystyle\sum_{k=1}^{\infty} \frac{k^3}{2^{\sqrt{k^3}}}$ converges.

b) $\displaystyle\sum_{k=1}^{\infty} \frac{k^k}{k!}$ ← use the Ratio Test; carefully justify algebra/limit calculations.

$$\lim_{k\to\infty} \frac{(k+1)^{(k+1)}/(k+1)!}{k^k/k!} = \lim_{k\to\infty} \frac{\cancel{(k+1)^{(k+1)}}^{\,(k+1)^k}\;\cancel{k!}}{k^k\;\cancel{(k+1)!}_{\,k+1}} = \lim_{k\to\infty}\left(\frac{k+1}{k}\right)^k = e$$

since $e>1$ the Ratio Test says that $\displaystyle\sum_{k=1}^{\infty} \frac{k^k}{k!}$ diverges.

c) $\displaystyle\sum_{k=1}^{\infty} \frac{k!}{(k+2)!}$ ← show Ratio Test is inconclusive and then use some other test.

$$\lim_{k\to\infty} \frac{(k+1)!/(k+3)!}{k!/(k+2)!} = \lim_{k\to\infty} \frac{\cancel{(k+1)!}^{\,k+1}\;\cancel{(k+2)!}}{\cancel{k!}\;\cancel{(k+3)!}_{\,k+3}} = \lim_{k\to\infty} \frac{k+1}{k+3} = 1$$

inconclusive.

another test: COMPARISON TEST

$$\frac{k!}{(k+2)!} = \frac{1}{(k+1)(k+2)} \leq \frac{1}{k^2}$$

$\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2}$ converges (p-series, $p>1$) so $\displaystyle\sum_{k=1}^{\infty} \frac{k!}{(k+2)!}$ must also converge.