236 TEST 1

You may use your notebook during the last half hour of this exam.
You may NOT use calculators, cell phones, loose papers, or peeking.

Math 236
February 1, 2011

Name: * Key *
By printing my name I pledge to uphold the honor code.

1. Use polynomial long division to write $\frac{x^2}{(x+3)(x-2)}$ as the sum of a polynomial and a proper rational function.

$\nwarrow x^2+x-6$

P.F #22

8 pts

$$\begin{array}{r} 1 \\ x^2+x-6 \overline{)\, x^2+0x+0} \\ \ominus x^2 \oplus x \ominus 6 \\ \hline -x+6 \end{array}$$

4 pts

$$\boxed{1+\frac{-x+6}{x^2+x-6}}$$

4 pts

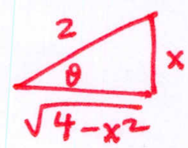
(obvious if you check $x^2=(x^2+x-6)+(-x+6)$.)

2. Write $\tan(\sin^{-1}\frac{x}{2})$ as an algebraic function. Show your work with a triangle.

8 pts

$\sin^{-1}\frac{x}{2}=\theta$

$\frac{x}{2}=\sin\theta$



6.3 example 4

So $\tan(\sin^{-1}\frac{x}{2}) = \tan\theta = \boxed{\frac{x}{\sqrt{4-x^2}}}$

3. Suppose you want to use the formula $\frac{M(b-a)^3}{24n^2}$ to find the smallest value of n for which we can guarantee that an n-rectangle Midpoint Sum for $\int_0^3 x^3\,dx$ will be within 0.1 of the exact answer. What is the numerical value of M in this example, and why?

8 pts

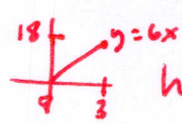
$M =$ max val. of $|f''(x)|$ on $[0,3]$

$f(x)=x^3$

$f'(x)=3x^2$

$f''(x)=6x$

6.7 #45



has max value of 18 on $[0,3]$

so $\boxed{M=18}$

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4. For each integral below, describe a method that will work but DO NOT SOLVE THE INTEGRAL HERE. Here are just a few examples of proper descriptions:

substitution with $u =$ ____ and $du =$ ____

parts with $u =$ ____, $du =$ ____, $v =$ ____, and $dv =$ ____

partial fractions decomposition of the form ______________ (do not solve for coefficients)

trig substitution with $x =$ ______ and $dx =$ ______

algebra/identity to rewrite as ______ and then ⟨describe method⟩

8 pts each (3+5)

$\boxed{6.2 \# 47}$

a) $\int \sqrt{x} \ln x \, dx$

parts with $u = \ln x$, $du = \frac{1}{x}dx$
$v = \frac{2}{3}x^{3/2}$, $dv = \sqrt{x}\,dx$

$\boxed{PF \# 34}$

b) $\int \frac{x^2 + 4x + 1}{x^3 + x^2} dx$

$x^2(x+1)$

partial fractions of form ~~[illegible]~~
$\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$

$\frac{Ax+B}{x^2}$

$\boxed{6.4 \# 51}$

c) $\int (1 - x^2)^{-\frac{3}{2}} dx$

$\frac{1}{(\sqrt{1-x^2})^3}$

trig sub with $x = \sin u$, $dx = \cos u \, du$

$\boxed{6.3 \# 58}$

d) $\int \tan x \cos^5 x \, dx$

$\frac{\sin x}{\cos x} \cos^5 x = \sin x \cos^4 x$

algebra to rewrite as $\sin x \cos^4 x$ and then usub with $u = \cos x$, $du = -\sin x \, dx$

$\boxed{6.1 \# 30}$

e) $\int \frac{x}{\sqrt{3x^2 + 1}} dx$

$(\sqrt{3}x)^2$

usub with
$u = 3x^2 + 1$, $du = 6x\,dx$
(or trig sub with $x = \frac{1}{\sqrt{3}} \tan u$, $dx = \frac{1}{\sqrt{3}} \sec^2 u \, du$)

$\boxed{6.3 \# 38}$

f) $\int \sin^5 x \cos^2 x \, dx$

$\sin^4 x \cos^2 x \sin x = (1 - \cos^2 x)^2 \cos^2 x \sin x$

algebra to rewrite as shown and then usub with $u = \cos x$, $du = -\sin x \, dx$

$\boxed{6.1 \# 54}$

g) $\int \frac{\sec^2 x}{\tan x + 1} dx$

usub with $u = \tan x + 1$, $du = \sec^2 x \, dx$

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5. Solve ONE of the integrals on the previous page, showing all work very clearly from start to finish. Choose the hardest one that you can solve correctly; you will get more points for solving something difficult than for solving something ~~easy~~.

a) $\int \sqrt{x} \ln x \, dx \quad \left(\begin{array}{l} u = \ln x \rightarrow du = \frac{1}{x} dx \\ v = \frac{2}{3} x^{3/2} \leftarrow dv = \sqrt{x}\, dx \end{array} \right) = \frac{2}{3} x^{3/2} \ln x - \int \frac{2}{3} x^{1/2} dx = \boxed{\frac{2}{3} x^{3/2} \ln x - \frac{4}{9} x^{3/2} + C}$

• b) $\int \frac{x^2+4x+1}{x^2(x+1)} dx = \int \left(\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1} \right) dx = \boxed{3 \ln|x| - x^{-1} - 2 \ln|x+1| + C}$

$\left(\begin{array}{l} Ax(x+1) + B(x+1) + Cx^2 = x^2 + 4x + 1 \\ \Rightarrow \ldots \Rightarrow A = 3, \ B = 1, \ C = -2 \end{array} \right)$

• c) $\int \frac{1}{(\sqrt{1-x^2})^3} dx \quad \left(\begin{array}{l} x = \sin u \\ dx = \cos u \, du \end{array} \right) = \int \frac{1}{(\cos u)^3} \cos u \, du = \int \sec^2 u \, du = \tan(\sin^{-1} x) + C$

$= \boxed{\frac{x}{\sqrt{1-x^2}} + C}$ (like #2)

• d) $\int \tan x \cos^5 x \, dx = \int \sin x \cos^4 x \, dx \quad \left(\begin{array}{l} u = \cos x \\ du = -\sin x \, dx \end{array} \right)$

$= -\int u^4 du = -\frac{1}{5} u^5 + C = \boxed{-\frac{1}{5} \cos^5 x + C}$

e) $\int \frac{x}{\sqrt{3x^2+1}} dx \quad \left(\begin{array}{l} u = 3x^2+1 \\ du = 6x \, dx \end{array} \right) = \int \frac{1}{6} \cdot \frac{1}{\sqrt{u}} du = \frac{1}{6} \cdot 2u^{1/2} + C = \boxed{\frac{1}{3} \sqrt{3x^2+1} + C}$

• f) $\int \sin^5 x \cos^2 x \, dx = \int (1 - \cos^2 x)^2 \cos^2 x \sin x \, dx \quad \left(\begin{array}{l} u = \cos x \\ du = -\sin x \, dx \end{array} \right)$

$= -\int (1-u^2)^2 u^2 du = -\int (u^2 - 2u^4 + u^6) du = \boxed{-\left(\frac{1}{3} \cos^3 x - \frac{2}{5} \cos^5 x + \frac{1}{7} \cos^7 x \right) + C}$

g) $\int \frac{\sec^2 x}{\tan x + 1} dx \quad \left(\begin{array}{l} u = \tan x + 1 \\ du = \sec^2 x \, dx \end{array} \right) = \int \frac{1}{u} du = \ln|u| + C = \boxed{\ln|\tan x + 1| + C}$

10 pts: (e), (g)   15 pts: (a)   20 pts: (b), (c), (d), (f)

Survey for 2 bonus points: How do you think you did? What is a question or topic that could have been on this exam, but wasn't?

definite integrals
simpson's rule
actually calculating a Riemann sum (Left, Mid, Trap, Simp, etc)
powers of sec x and tan x
prove integrals of tan x, sec x, ln x
completing the square

up to 20

sCRAP

(I will not be grading anything on the scrap page)