236 TEST 2

You may use your notebook during the last half hour of this exam.
You may NOT use calculators, cell phones, loose papers, or peeking.

Math 236
March 1, 2011

Name: * key *

By printing my name I pledge to uphold the honor code.

1. True or false?

T F L'Hôpital's Rule only applies to limits where $x \to 0$ or $x \to \infty$.

T F If $\lim_{x\to 2} \ln(f(x)) = -\infty$, then $\lim_{x\to 2} f(x) = -\infty$.

T F $\displaystyle\lim_{x\to\infty} \frac{\sqrt{x}}{125 \ln x} = \infty$.

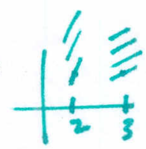
T F $f(x) = 2x^{100}$ dominates $g(x) = 100(2^x)$ as $x \to \infty$.

3pts each

T F $y(t) = \sqrt{t+9}$ is a solution to the differential equation $\frac{dy}{dt} = \frac{1}{2y}$.

$y = \sqrt{t+9}$
$y' = \frac{1}{2}(t+9)^{-1/2} = \frac{1}{2y}$

T F In a slopefield for a differential equation of the form $\frac{dy}{dx} = g(x)$, the slope at $(2, b)$ will be the same as the slope at $(3, b)$.



T F If $y_1(x)$ and $y_2(x)$ are both solutions to ~~the~~ a differential equation $\cancel{\frac{dy}{dx} = dy}$, then the sum $y_1(x) + y_2(x)$ is also a solution to ~~the~~ that differential equation.

T F If $\frac{dP}{dt} = kP(1 - \frac{P}{500})$, then for small values of t the population $P(t)$ behaves similarly to an exponential model.

T F If $f(x)$ is a positive-valued function and $\int_3^\infty f(x)\, dx$ diverges, then $\int_3^\infty (2f(x) + 1)\, dx$ also diverges.

$2f(x)+1 \geq f(x)$
since $f(x) \geq 0$

T F If $f(x)$ is a positive-valued function and $\int_0^1 f(x)\, dx$ converges, and if $g(x) \geq f(x)$ for all x, then $\int_0^1 g(x)\, dx$ diverges.

$\geq$ finite $\not\Rightarrow \infty$

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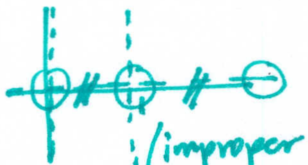
2. Setting up integrals: Express each of the following in terms of proper definite integrals. Put boxes around your final answers.

PLEASE DO NOT SOLVE THE INTEGRALS.

8 pts each

a) the area between the graph of $f(x) = \frac{1}{x-x^2}$ and the x-axis on $[0, \infty)$ * *proper*

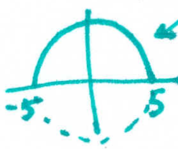
$x(1-x)$



(improper at $x=0$, $x=1$, $x\to\infty$)

$$\boxed{\lim_{A\to 0^+}\int_A^{1/2}\frac{1}{x-x^2}\,dx + \lim_{B\to 1^-}\int_{1/2}^{B}\frac{1}{x-x^2}\,dx + \lim_{C\to 1^+}\int_C^{2}\frac{1}{x-x^2}\,dx + \lim_{D\to\infty}\int_2^{D}\frac{1}{x-x^2}\,dx}$$

b) the circumference of a circle of radius 5



$x^2+y^2=5^2$

$y=\sqrt{25-x^2}$

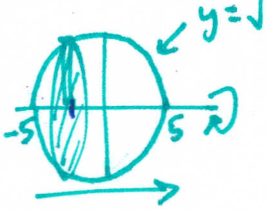
C = twice top arc length

$y' = \frac{1}{2}(25-x^2)^{-1/2}(-2x)$

$$\boxed{2\int_{-5}^{5}\sqrt{1+\left(\frac{-x}{\sqrt{25-x^2}}\right)^2}\,dx}$$

(or 4 times integral from 0 to 5)

c) the volume of a sphere of radius 5, with the disc method



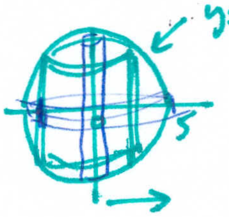
$y=\sqrt{25-x^2}$

rep. disk at x_k^* has radius $\sqrt{25-(x_k^*)^2}$

$$\boxed{\pi\int_{-5}^{5}(25-x^2)\,dx}$$

(or, equivalently, with y.)

d) the volume of a sphere of radius 5, with the shell method



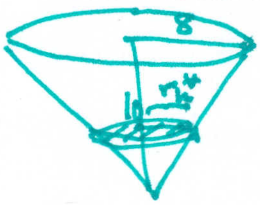
$y=\sqrt{25-x^2}$

rep. shell at x_k^* has radius x_k^* and height $2\sqrt{25-(x_k^*)^2}$

$$\boxed{2\pi\int_{0}^{5}x\cdot 2\sqrt{25-x^2}\,dx}$$

(also same w/y)

e) the work required to pump all the water out of the top of a upright conical tank that is 10 feet high and has a radius of 8 feet at the top



$y=10$

y_k^*

$y=0$

similar △'s says $\frac{r_k^*}{y_k^*}=\frac{8}{10}$

$$\boxed{62.4\,\pi\left(\frac{4}{5}\right)^2\int_0^{10}y^2(10-y)\,dy}$$

work to lift slice at y_k^* is

$W = F\cdot d = w\cdot V\cdot d = (62.4\ \text{lbs}/\text{ft}^3)\,\pi\left(\frac{8}{10}y_k^*\right)^2\Delta y\,(10-y_k^*)$

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$$\left\{ \begin{array}{l} \sin^2 u + \cos^2 u = 1 \\ 1 + \cot^2 u = \csc^2 u \end{array} \right.$$

3. Calculations: Show all work and put a box around your final answer.

a) find $\displaystyle\int \frac{1}{x^4\sqrt{4-x^2}}\,dx$

$$\left(\begin{array}{l} x = 2\sin u \\ dx = 2\cos u\,du \end{array} \right) = \int \frac{2\cos u}{16\sin^4 u\sqrt{4-4\sin^2 u}}\,du$$

$$= \frac{1}{8}\int \frac{\cos u}{\sin^4 u\,(2\cancel{\cos u})}\,du = \frac{1}{16}\int \csc^4 u\,du$$

$$= \frac{1}{16}\int \csc^2 u\,(1+\cot^2 u)\,du = \frac{1}{16}\int \csc^2 u\,du + \frac{1}{16}\int \csc^2 u \cot^2 u\,du$$

$$\left(\begin{array}{l} w = \cot u \\ dw = -\csc^2 u\,du \end{array} \right)$$

$$= \frac{1}{16}(-\cot u) + \frac{1}{16}(-1)\int w^2\,dw$$

15 pts each

$$= -\frac{1}{16}\cot u - \frac{1}{16}\cdot\frac{1}{3}w^3 + C = -\frac{1}{16}\cot u - \frac{1}{16(3)}\cot^3 u + C$$

$$= \boxed{-\frac{1}{16}\,\frac{\sqrt{4-x^2}}{x} - \frac{1}{16\cdot 3}\left(\frac{\sqrt{4-x^2}}{x}\right)^3 + C}$$

$$\left\{ \begin{array}{l} x = 2\sin u \\ \frac{x}{2} = \sin u \end{array} \right.$$

$$\cot u = \frac{\text{adj}}{\text{opp}}$$

b) solve $\displaystyle\frac{dy}{dx} = 0.5y(2-y)$

$$\int \frac{1}{0.5y(2-y)}\,dy = \int dx$$

$$\frac{1}{0.5}\int \left(\frac{A}{y} + \frac{B}{2-y}\right)dy = \int dx$$

$$\left\{ \begin{array}{l} A(2-y) + By = 1 \\ \Rightarrow B\cdot 2 = 1 \\ \quad A\cdot 2 = 1 \\ \Rightarrow A = 1/2,\ B = 1/2 \end{array} \right.$$

$$\frac{1}{\cancel{1/2}}\int \left(\frac{\cancel{1/2}}{y} + \frac{\cancel{1/2}}{2-y}\right)dy = \int dx$$

$$\int \left(\frac{1}{y} + \frac{1}{2-y}\right)dy = \int dx$$

$$\ln|y| + (-1)\ln|2-y| = x + C$$

$$\ln\left|\frac{y}{2-y}\right| = x + C$$

$$\frac{y}{2-y} = e^{x+C} = Ae^x$$

$$y = Ae^x(2-y)$$

$$y = 2Ae^x - yAe^x$$

$$y + yAe^x = 2Ae^x$$

$$y(1+Ae^x) = 2Ae^x$$

$$\boxed{y = \frac{2Ae^x}{1+Ae^x}}$$

Bonus Survey: How did you do? What could have been on this exam, but wasn't?

You said:

Newton's Law of cooling
volumes after rotating around non-axis lines
Riemann sums for arc length, volumes, physics approx.
exponential or logistic growth
washers

Calculate an improper integral or determine converge/diverge
calculate w/ L'H
mass
hydrostatic force

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sCRAP

(I will not be grading anything on the scrap page)