236 TEST 2

You may use your notebook during the last half hour of this exam.
You may NOT use calculators, cell phones, loose papers, or peeking.

Math 236
March 1, 2011

Name: ____________________________

By printing my name here, I pledge to uphold the honor code.

1. True or false?

T (F) L'Hôpital's Rule only applies to limits where \( x \to 0 \) or \( x \to \infty \).

T (F) If \( \lim_{x \to 2} \ln(f(x)) = -\infty \), then \( \lim_{x \to 2} f(x) = -\infty \).

T (F) \( \lim_{x \to \infty} \frac{\sqrt{x}}{125 \ln x} = \infty \).

T (F) \( f(x) = 2x^{100} \) dominates \( g(x) = 100(2^x) \) as \( x \to \infty \).

T (F) \( y(t) = \sqrt{t+9} \) is a solution to the differential equation \( \frac{dy}{dt} = \frac{1}{2y} \).

T (F) In a slopefield for a differential equation of the form \( \frac{dy}{dx} = g(x) \), the slope at \( (2, b) \) will be the same as the slope at \( (3, b) \).

T (F) If \( y_1(x) \) and \( y_2(x) \) are both solutions to the differential equation \( \frac{dy}{dx} = g(x) \), then the sum \( y_1(x) + y_2(x) \) is also a solution to the differential equation.

T (F) If \( \frac{dP}{dt} = kP(1 - \frac{P}{500}) \), then for small values of \( t \) the population \( P(t) \) behaves similarly to an exponential model.

T (F) If \( f(x) \) is a positive-valued function and \( \int_{3}^{\infty} f(x) \, dx \) diverges, then \( \int_{3}^{\infty} (2f(x) + 1) \, dx \) also diverges.

T (F) If \( f(x) \) is a positive-valued function and \( \int_{0}^{1} f(x) \, dx \) converges, and if \( g(x) \geq f(x) \) for all \( x \), then \( \int_{0}^{1} g(x) \, dx \) diverges.

30 pts each
2. Setting up integrals: Express each of the following in terms of proper definite integrals. Put boxes around your final answers.

PLEASE DO NOT SOLVE THE INTEGRALS.

a) the area between the graph of \( f(x) = \frac{1}{x-x^2} \) and the \( x \)-axis on \((0, \infty)\)

\[
\lim_{A \to 0^+} \int_0^A \frac{1}{x-x^2} \, dx + \lim_{B \to 1^-} \int_A^B \frac{1}{x-x^2} \, dx
+ \lim_{D \to 0^-} \int_D^1 \frac{1}{x-x^2} \, dx
\]

b) the circumference of a circle of radius 5

\( C = \text{twice the arc length} \)

\[ y = \frac{1}{2} (25-x^{1/2}) (2x) \]

\[ 2 \int_{-5}^{5} \sqrt{1 + \left( \frac{-x}{\sqrt{25- x^2}} \right)^2} \, dx \]

(or 4 times integral from 0 to 5)

(c) the volume of a sphere of radius 5, with the disc method

\[ \pi \int_{-5}^{5} (25 - x^2) \, dx \]

(abbreviated with \( y \))

(d) the volume of a sphere of radius 5, with the shell method

\[ 2 \pi \int_0^5 x \cdot 2 \sqrt{25-x^2} \, dx \]

(also same as \( y \))

(e) the work required to pump all the water out of the top of a upright conical tank that is 10 feet high and has a radius of 8 feet at the top

\[ 62.4 \pi \left( \frac{4}{3} \right)^2 \int_0^{10} y^2 (10-y) \, dy \]

Work to lift slice at \( y^* \) is

\[ W = F \cdot d = w \cdot V \cdot d = \left( \frac{62.4 \; \text{lb}}{\text{ft}^3} \right) \pi \left( \frac{8}{10} y^* \right)^2 dy \cdot (10-y^*) \]
3. Calculations: Show all work and put a box around your final answer.

a) find \( \int \frac{1}{x^4 \sqrt{4-x^2}} \, dx \)

\( (x = 2 \sin u \quad dx = 2 \cos u \, du) = \int \frac{2 \cos u}{16 \sin^4 u \sqrt{4-4 \sin^2 u}} \, du \)

\[ \frac{\cos u}{8 \sin^4 u} \, du = \frac{1}{16} \int \csc^4 u \, du \]

\[ \frac{1}{16} \int \csc^2 u (1 + \cot^2 u) \, du = \frac{1}{16} \int \csc^2 u \, du + \frac{1}{16} \int \csc^2 u \cot u \, dw \]

\[ \frac{1}{16} \cot u - \frac{1}{16} \cdot \frac{1}{3} \cdot w^3 + C = \frac{1}{16} - \frac{1}{16(3)} \cot^3 u + C \]

\[ = -\frac{1}{16} \frac{\sqrt{4-x^2}}{x} - \frac{1}{16 \cdot 3} \left( \frac{\sqrt{4-x^2}}{x} \right)^3 + C \]

b) solve \( \frac{dy}{dx} = 0.5y(2 - y) \)

\[ \frac{1}{0.5} \int \frac{1}{y(2-y)} \, dy = \int \, dx \]

\[ \frac{1}{2} \int \left( \frac{1}{y} + \frac{1}{2-y} \right) \, dy = \int \, dx \]

\[ \ln |y| + \ln |2-y| = x + C \]

\[ \ln |y(2-y)| = x + C \]

\[ y = 2A e^x \quad y(2-y) = 2A e^x \]

\[ y = 2A e^x - yA e^x \]

\[ y = \frac{2A e^x}{1+yA} \]

**Bonus Survey:** How did you do? What could have been on this exam, but wasn’t?

**You Said:**

- Newton’s Law of cooling
- Volumes after rotating around non-axis lines
- Riemann sums for arc length, volumes, physics approx
- Exponential or logistic growth
- Calculate an improper integral or determine convergence/divergence
- Calculate w/L’Hôpital’s rule
- Mass, hydrostatic force
sCRAP

(I will not be grading anything on the scrap page)