# 236 TEST 3

*You may use your notebook during the last half hour of this exam.*
*You may NOT use calculators, cell phones, loose papers, or peeking.*

Math 236
April 5, 2011

**Name:** _____

By printing my name I pledge to uphold the honor code.

**1.** True or false?

**T** **F** Every monotonic sequence of real numbers has either a greatest lower bound, a greatest lower bound, or both.

**T** **F** Every convergent sequence of real numbers is bounded.

**T** **F** Every bounded sequence of real numbers is convergent.

**T** **F** If $\sum_{k=1}^{\infty} a_k$ diverges, then $\{a_k\}$ diverges.

**T** **F** If $\{a_k\}$ converges to 0, then $\sum_{k=1}^{\infty} a_k$ converges.

**T** **F** If $\sum_{k=1}^{\infty} a_k$ converges and $\{S_n\}$ is its sequence of partial sums, then $\{S_n\}$ converges.

**T** **F** $\sum_{k=1}^{\infty} (-1)^k a_k$ converges if and only if $\sum_{k=1}^{\infty} (-1)^{k+1} a_k$ converges.

**T** **F** If $\sum_{k=1}^{\infty} a_k$ converges conditionally, then $\sum_{k=1}^{\infty} |a_k|$ diverges.

**T** **F** If $\sum_{k=1}^{\infty} a_k$ and $\sum_{k=1}^{\infty} b_k$ both converge, then $\lim_{k\to\infty} \frac{a_k}{b_k}$ is finite.

**2.** Circle EVERY description that applies to each sequence or series. You do not need to show any work or describe the tests that you used.

| | | | | |
|---|---|---|---|---|
| $\left\{\frac{k^2}{(k+1)^2}\right\}$ | (monotonic) | (bounded) | (convergent) | (divergent) |
| $\left\{\frac{(-1)^k}{k-1}\right\}$ | (monotonic) | (bounded) | (convergent) | (divergent) |
| $\left\{\sin(\frac{1}{k!})\right\}$ | (monotonic) | (bounded) | (convergent) | (divergent) |
| $\sum_{k=1}^{\infty} \ln\left(\frac{1}{k^2+1}\right)$ | (convergent) | (divergent) | | |
| $\sum_{k=1}^{\infty} \left(\frac{k}{k+1}\right)^{k^2}$ | (convergent) | (divergent) | | |
| $\sum_{k=1}^{\infty} \frac{\sqrt{k}}{k^2-3}$ | (convergent) | (divergent) | | |
| $\sum_{k=1}^{\infty} \frac{1}{k(\ln k)^2}$ | (convergent) | (divergent) | | |
| $\sum_{k=1}^{\infty} \frac{2^k}{k!}$ | (convergent) | (divergent) | | |
| $\sum_{k=1}^{\infty} \frac{(-1)^k\, k^2}{k!}$ | (abs. conv.) | (cond. conv.) | (divergent) | |
| $\sum_{k=1}^{\infty} \cos^3(\frac{1}{k})$ | (abs. conv.) | (cond. conv.) | (divergent) | |
| $\sum_{k=1}^{\infty} \frac{\cos(\pi k)}{k}$ | (abs. conv.) | (cond. conv.) | (divergent) | |

**3.** Find the exact sum of each of the following series. Show your work very carefully.

**a)** $\displaystyle\sum_{k=3}^{\infty} \frac{2^{k+2}}{3^k}$

**b)** $\displaystyle\sum_{k=1}^{\infty} \frac{9}{k^2+3k}$

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**Bonus Survey:** How did you do? What could have been on this exam, but wasn't?

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sCRAP (I will not be grading anything on the scrap page)