1. True or false?

T  F  Every monotonic sequence of real numbers has either a greatest lower bound, a greatest lower bound, or both.

T  F  Every convergent sequence of real numbers is bounded.

T  F  Every bounded sequence of real numbers is convergent.

T  F  If \( \sum_{k=1}^{\infty} a_k \) diverges, then \( \{a_k\} \) diverges.

T  F  If \( \{a_k\} \) converges to 0, then \( \sum_{k=1}^{\infty} a_k \) converges.

T  F  If \( \sum_{k=1}^{\infty} a_k \) converges and \( \{S_n\} \) is its sequence of partial sums, then \( \{S_n\} \) converges.

T  F  \( \sum_{k=1}^{\infty} (-1)^k a_k \) converges if and only if \( \sum_{k=1}^{\infty} (-1)^{k+1} a_k \) converges.

T  F  If \( \sum_{k=1}^{\infty} a_k \) converges conditionally, then \( \sum_{k=1}^{\infty} |a_k| \) diverges.

T  F  If \( \sum_{k=1}^{\infty} a_k \) and \( \sum_{k=1}^{\infty} b_k \) both converge, then \( \lim_{k \to \infty} \frac{a_k}{b_k} \) is finite.
2. Circle EVERY description that applies to each sequence or series. You do not need to show any work or describe the tests that you used.

\[
\left\{ \frac{k^2}{(k + 1)^2} \right\} \quad \text{(monotonic)} \quad \text{(bounded)} \quad \text{(convergent)} \quad \text{(divergent)}
\]

\[
\left\{ \frac{(-1)^k}{k - 1} \right\} \quad \text{(monotonic)} \quad \text{(bounded)} \quad \text{(convergent)} \quad \text{(divergent)}
\]

\[
\left\{ \sin\left(\frac{1}{k^2}\right) \right\} \quad \text{(monotonic)} \quad \text{(bounded)} \quad \text{(convergent)} \quad \text{(divergent)}
\]

\[
\sum_{k=1}^{\infty} \ln\left(\frac{1}{k^2 + 1}\right) \quad \text{(convergent)} \quad \text{(divergent)}
\]

\[
\sum_{k=1}^{\infty} \left(\frac{k}{k + 1}\right)^k \quad \text{(convergent)} \quad \text{(divergent)}
\]

\[
\sum_{k=1}^{\infty} \frac{\sqrt{k}}{k^2 - 3} \quad \text{(convergent)} \quad \text{(divergent)}
\]

\[
\sum_{k=1}^{\infty} \frac{1}{k(\ln k)^2} \quad \text{(convergent)} \quad \text{(divergent)}
\]

\[
\sum_{k=1}^{\infty} \frac{2^k}{k!} \quad \text{(convergent)} \quad \text{(divergent)}
\]

\[
\sum_{k=1}^{\infty} \frac{(-1)^k k^2}{k!} \quad \text{(abs. conv.)} \quad \text{(cond. conv.)} \quad \text{(divergent)}
\]

\[
\sum_{k=1}^{\infty} \cos^3\left(\frac{1}{k}\right) \quad \text{(abs. conv.)} \quad \text{(cond. conv.)} \quad \text{(divergent)}
\]

\[
\sum_{k=1}^{\infty} \frac{\cos(\pi k)}{k} \quad \text{(abs. conv.)} \quad \text{(cond. conv.)} \quad \text{(divergent)}
\]
3. Find the exact sum of each of the following series. Show your work very carefully.

a) \[ \sum_{k=3}^{\infty} \frac{2^{k+2}}{3^k} \]

b) \[ \sum_{k=1}^{\infty} \frac{9}{k^2 + 3k} \]

Bonus Survey: How did you do? What could have been on this exam, but wasn’t?
sCRAP

(I will not be grading anything on the scrap page)