

Determine whether each sequence or series converges or diverges. If a sequence converges, determine what it converges to. If a series converges, find its exact sum if it is possible to do so; otherwise, give an approximation of its sum. (*Hint: There are three sums you can find exactly. One involves telescoping, one involves geometric series, one is related to the alternating harmonic series.*) If a series has any negative terms, determine both absolute and conditional convergence.

1. $\left\{ \frac{2^k}{k!} \right\}$

2. $\left\{ \frac{k^{\frac{1}{k}}}{k} \right\}$

3. $\left\{ \frac{2^{\frac{1}{k}}}{3^{\frac{1}{k}}} \right\}$

4. $\left\{ \frac{\ln k}{k^2} \right\}$

5. $\left\{ \left(\frac{1}{2}\right)^k - \left(\frac{1}{3}\right)^k \right\}$

6. $\sum_{k=1}^{\infty} \frac{2^k}{k!}$

7. $\sum_{k=1}^{\infty} \frac{k^{\frac{1}{k}}}{k}$

8. $\sum_{k=1}^{\infty} \frac{2^{\frac{1}{k}}}{3^{\frac{1}{k}}}$

9. $\sum_{k=1}^{\infty} \frac{\ln k}{k^2}$

10. $\sum_{k=1}^{\infty} \left(\left(\frac{1}{2}\right)^k - \left(\frac{1}{3}\right)^k \right)$

11. $\sum_{k=1}^{\infty} \frac{1}{k(k+3)}$

12. $\sum_{k=1}^{\infty} \frac{\sqrt{k+1}}{k^2+1}$

13. $\sum_{k=1}^{\infty} \left(1 - \frac{1}{k}\right)^{k^2}$

14. $\sum_{k=1}^{\infty} \left(\frac{1}{2k} - \frac{1}{2k+1} \right)$

15. $\sum_{k=1}^{\infty} \frac{k!}{k^2 + k!}$

16. $\sum_{k=1}^{\infty} (-1)^k k e^{-k}$

17. $\sum_{k=1}^{\infty} \frac{(2k)!}{(k!)^2}$

18. $\sum_{k=1}^{\infty} \frac{\cos k}{k^3}$

19. $\sum_{k=1}^{\infty} \frac{2 \cdot 5 \cdot 8 \cdots (3k-1)}{3 \cdot 6 \cdot 9 \cdots (3k)}$

20. $\sum_{k=1}^{\infty} \frac{(-1)^k \ln k}{k}$