Math 245-99 02/14/73

Writeup of Problem 8.8 Wiggy Wince (W) and Pinky Prince (P)

Your first LaTeX assignment is to use LaTeX to produce a document that replicates this one as exactly as possible, with just two differences: First, replace the class section number, date, and names above with your own. Second, make the following letter substitutions so that I know that you did not just photocopy this document: in the first two proofs, change each m to n; in the last proof, change each c to b. Your grade on this assignment will be based on how much your paper looks like this one.

Problem:

Prove that every integer that is divisible by 6 is even.

Solution:

Proof. Suppose $m \in \mathbb{Z}$ is divisible by 6. Then there is some $k \in \mathbb{Z}$ such that $m = 6k$. Therefore $m = 2(3k)$, and since $3k$ is also in $\mathbb{Z}$, this means that m is divisible by 2 and therefore that m is even. $\square$

Problem:

Define $A = \{m \in \mathbb{Z} \mid m^3 - m^2 - 6m = 0\}$. Prove that if $m \in A$ then $m = -2$, 0, or 3.

Solution:

Proof. Let $A = \{m \in \mathbb{Z} \mid m^3 - m^2 - 6m = 0\}$. Note that

$$\begin{aligned} m^3 - m^2 - 6m &= m(m^2 - m - 6) && \text{(factor out an } m\text{)} \\ &= m(m+2)(m-3). && \text{(factor the quadratic)} \end{aligned}$$

Therefore if $m \in A$ then $m(m+2)(m-3) = 0$, which means that m must be equal to one of -2, 0, or 3. $\square$

Problem:

Prove that if $a, c \in \mathbb{R}$ with $a \leq c$ then $[c, \infty) \subseteq [a, \infty)$.

Solution:

Proof. Suppose $a \leq c$ in $\mathbb{R}$. For all $x \in \mathbb{R}$,

$$\begin{aligned} x \in [c, \infty) &\Longrightarrow x \geq c \\ &\Longrightarrow x \geq c \geq a && (c \geq a \text{ by hypothesis}) \\ &\Longrightarrow x \geq a && \text{(transitivity)} \\ &\Longrightarrow x \in [a, \infty). \end{aligned}$$

Therefore we have $[c, \infty) \subseteq [a, \infty)$. $\square$