Math 245-00 9/23/09

Writeup of Problem 6.8 Walter Writer (W) and Percy Presenter (P)

This sample document provides a template for writing a LaTeX document suitable for homework assignments in Laura's 245 class. Compare what is in the typeset version of this document to the file `latexsample.tex`. Note in particular that anything typed after a percent sign in the text file is treated as a comment and is ignored by the compiler. Comments in the text file refer both to LaTeX and to hints about writing good solutions and proofs.

The very basics of LaTeX:
(compare the typeset document latexsample.pdf and the text file latexsample.tex)

- Extra spaces in the text file do not appear in the typeset document.

  Except for a double carriage-return, that makes a new line. Single carriage returns don't do anything.

- Mathematical expressions are typed between dollar signs like this: $y = x^2 + 1$. To make a centered equation on its own line, use double dollar signs, like this:

$$y = x^2 + 1.$$

  The percent sign above the equation in the text file just makes it so that extra space is not added between the centered equation and the main paragraph above.

- Many LaTeX commands and math symbols start with a backslash symbol. For example, $\sin x$ and $\{x \in \mathbb{R} \mid x \geq 0\}$. Notice that the set-notation parentheses need to have backslashes before them (while regular parentheses do not). This is because in LaTeX, those squiggly parentheses often have other uses.

- If you need to put something in italics you do it *like this*. Or maybe you need to have something in **boldface**. Or maybe ***both***.

- Notice that to have quotes appear "correctly" in the typeset document you may have to type them yourself using the ` and ' keys instead of the " key.

- Here is some random notation you might need (again, look at the text document):

  x_2, x_{25}, x^2, x^{25}, ± 4, $x \neq 17$, $x > 5$, $x < 5$, $x \geq 5$, $x \leq 5$, $\{1, 2, 3\}$, $\{x \mid \sqrt{x} > 2\}$, ∞.

  $A \subset B$, $A \subseteq B$, $A \not\subset B$, $A \not\subseteq B$, $A \setminus B$, A^c, $A \cap B$, $A \cup B$, $x \in A$, $x \notin A$, $|A|$, $\mathcal{P}(A)$, $\emptyset$.

  $\frac{5}{1+x}$, $\dfrac{5}{1+x}$, $\bigcap_{i=1}^n S_i$, $\displaystyle\bigcap_{i=1}^n S_i$, $\bigcup_{i=1}^n S_i$, $\displaystyle\bigcup_{i=1}^n S_i$, $\sum_{k=1}^{10} a_k$, $\displaystyle\sum_{k=1}^{10} a_k$, $\prod_{k=1}^{10} a_k$, $\displaystyle\prod_{k=1}^{10} a_k$.

  $\mathbb{R}$, $\mathbb{Q}$, $\mathbb{Z}$, $\mathbb{N}$, $\clubsuit$, $\diamondsuit$, $\heartsuit$, $\spadesuit$, $\rightarrow$, $\leftarrow$, $\leftrightarrow$, $\longrightarrow$, $\longleftarrow$, $\longleftrightarrow$, $\Rightarrow$, $\Leftarrow$, $\Leftrightarrow$, $\Longrightarrow$, $\Longleftarrow$, $\Longleftrightarrow$, $\mapsto$, $\longmapsto$.

  $\mathcal{P}$, $\mathcal{S}$, $\mathcal{F}$, $\forall$, $\exists$, $\vee$, $\wedge$, $\neg$, $\sim$, $\approx$, $\equiv$, $\times$, $*$, $\star$, $|$, $a|b$, $|x|$, $\|x\|$, $\lceil x \rceil$, $\lfloor x \rfloor$, $\{x \in \mathbb{Z} \mid x \text{ is prime}\}$.

  gcd, lcm, $\binom{n}{k}$, $\binom{n+1}{k}$, $a = \binom{n+1}{k}$, $\prec$, $\preceq$, $\succ$, $\succeq$, $f\colon [0, \infty) \to \mathbb{R}$, $f \circ g$ {, }, $, %, &, _, #.

- For how to write other symbols or to look up error messages that you get in your .log file, use teh google tubes.

- On the next page we write up three sample problems so you can see how things might work out when you try to write up your homework.

Problem:

42 students are taking algebra, 32 are taking Spanish, 7 are taking both.
How many students are there?

Solution:

The number of students can be found by adding up the number of students who are taking algebra and the number of students who are taking Spanish, and then subtracting off the number of students who are taking both (since they would otherwise be counted twice):

$$43 + 32 - 7 = 68 \text{ students.}$$

Problem:

P and Q are statements.
Show that $\neg Q$ and $\neg(P \wedge Q) \wedge \neg Q$ are logically equivalent.

Solution:

P	Q	$P \wedge Q$	$\neg(P \wedge Q)$	$\neg Q$	$\neg(P \wedge Q) \wedge \neg Q$
T	T	T	F	F	F
T	F	F	T	T	T
F	T	F	T	F	F
F	F	F	T	T	T

Since the truth-values for $\neg Q$ and $\neg(P \wedge Q) \wedge \neg Q$ are the same for all possible truth-values of P and Q, the two statements are logicaly equivalent.

Problem:

Prove that $n^3 + n$ is even for every integer n.

Solution:

Proof. Suppose n is any integer. We will examine two cases:

If n is even, then $n = 2k$ for some $k \in \mathbb{Z}$, and therefore:

$$\begin{aligned} n^3 + n &= (2k)^3 + (2k) && \text{(since } n = 2k\text{)} \\ &= 8k^3 + 2k \\ &= 2(4k^3 + k). && \text{(factor out a 2)} \end{aligned}$$

Since $4k^3 + k \in \mathbb{Z}$, this means $n^3 + n$ is divisible by 2 and therefore is even.

On the other hand, suppose n is odd. Then $n = 2k + 1$ for some $k \in \mathbb{Z}$, and thus:

$$\begin{aligned} n^3 + n &= (2k+1)^3 + (2k+1) && \text{(since } n = 2k+1\text{)} \\ &= (8k^3 + 12k^2 + 6k + 1) + (2k+1) && \text{(multiply out)} \\ &= 8k^3 + 12k^2 + 8k + 2 \\ &= 2(4k^3 + 6k^2 + 4k + 1). && \text{(factor out a 2)} \end{aligned}$$

Once again, $n^3 + n$ is a multiple of 2 and therefore is even. □