Math 353 Test 1

January 25, 2013 Name: * key *

1. Prove by induction that for all $n \in \mathbb{N}$ the sequence

$$n, n, n-1, n-1, \ldots, 3, 3, 2, 2, 1, 1$$

is graphic. Arrange your steps as outlined below, and include all quantifiers and necessary logical words in your argument.

Base case: When $n = 1$, ... the sequence is 1,1 , which is graphic: •—•

Inductive hypothesis: Assume that... for a particular $n \in \mathbb{N}$, the sequence $n, n, n-1, n-1, \ldots, 1, 1$ is graphic.

Inductive step: We will show that... the sequence $n+1, n+1, n, n, \ldots, 1, 1$ is graphic.

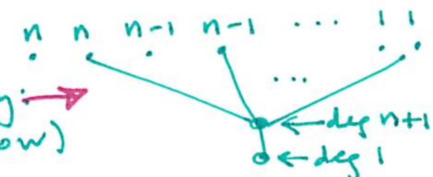
(now show it)

20pts

by the ind. hyp. $\exists$ a graph with sequence

add 2 vxs to this graph, connecting this way:

(to one vx of each degree and then below)

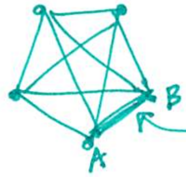


this new graph has degrees $n+1, n+1, n, n, \ldots, 1, 1$,

WWWWW.

2. Argue that each edge of K_5 is part of exactly fifteen cycles. Make sure that your counting argument is clear (don't just list/draw the cycles).

10 pts



K_5 is completely symmetric and each edge has the same adjacency properties.

WLOG look at this edge.

cycles of length 3: $\underset{1}{A} \to \underset{1}{B} \to \underset{3}{X} \to \underset{1}{A}$ $\Rightarrow$ 3 ways

cycles of length 4: $\underset{1}{A} \to \underset{1}{B} \to \underset{3}{X} \to \underset{2}{Y} \to \underset{1}{A}$ $\Rightarrow$ 6 ways

cycles of length 5: $\underset{1}{A} \to \underset{1}{B} \to \underset{3}{X} \to \underset{2}{Y} \to \underset{1}{Z} \to \underset{1}{A}$ $\Rightarrow$ 6 ways

15 ways