# IN-CLASS TEST I

Math 430
September 17, 2004

**Name:** ______________________________
By writing my name I swear by the honor code.

**Read all of the following information before starting the exam:**

- Your work will be graded for clarity as well as for mathematical accuracy. Make sure that your logic is clear and that I can tell what you are proving, and how. Provide reasons for steps whenever possible.
- Don't get hung up on any one problem. If you get stuck, move on and come back to the problem later.
- By writing your name above, you agree to the JMU honor code. In particular, this means that you may not use any notes or crib sheets during this exam, that all work must be your own, and that you may not obtain advance information revealing the problems on this exam.
- This test has 4 multi-part problems and is worth 100 points, plus some extra credit at the end. Make sure that you have all of the pages!
- Good luck!

**1.** (*9 pts*) **True/False.** Determine whether each statement is true (T) or false (F).

**a.** (*3 pts*) **T** **F** The sets $\mathbb{Z}$ and $\mathbb{Q}$ have the same cardinality.

**b.** (*3 pts*) **T** **F** Every group with three elements is abelian.

**c.** (*3 pts*) **T** **F** A binary operation $*$ on a set $S$ is commutative if there exist $a, b \in S$ such that $a * b = b * a$.

**2.** (*30 pts*) **Short Answer.** Fill in the blanks or give short answers, as appropriate.

**a.** (*6 pts*) If $\phi\colon U_7 \to \mathbb{Z}_7$ is an isomorphism with the property that $\phi(\zeta) = 4$, then $\phi(\zeta^3)$ must be equal to ______ .

**b.** (*6 pts*) List all possible partitions of the set $S = \{a, b, c\}$.

**c.** (*6 pts*) Give an example of a group with four elements that is *not* isomorphic to $\mathbb{Z}_4$. (Describe your group with a table.)

**d.** (*6 pts*) Give an example of a non-abelian group.

**e.** (*6 pts*) Draw a directed graph representing a relation $\mathcal{R}$ on $X = \{\alpha, \beta, \gamma\}$ that is reflexive and symmetric, but not transitive.

**3.** (*37 pts*) **Proofs.** You will be graded on the accuracy *and the clarity* of your arguments.

**a.** (*18 pts*) Suppose $X$ and $Y$ are sets and $f\colon X \to Y$ is a function. Consider the relation on $X$ defined by $a\,\mathcal{R}\,b$ if $f(a) = f(b)$. Prove that $\mathcal{R}$ is an equivalence relation.

**b.** (*18 pts*) Use the group axioms to prove that the left cancellation law holds in every group $\langle G, * \rangle$. Be sure you show and provide reasons for all steps in your proof.

**c.** (*1 pt*) Write whatever you like in this box for one point: $\boxed{\phantom{xxxxxxxxxxxxxxxx}}$

**4.** (*24 pts*) **Why Not?** Make sure your justifications are clear, convincing, and complete. (For example, to show that a binary algebraic structure is not a group, it would *not* be enough simply to write "not associative;" you would also have to explain *why* you know it fails to be associative.)

**a.** (*6 pts*) Define $a * b = a - b$ for all $a$ and $b$ in $\mathbb{Z}^+$. Explain why $*$ is *not* a binary operation on $\mathbb{Z}^+$.

**b.** (*6 pts*) Suppose $\phi\colon \langle \mathbb{Z}, + \rangle \to \langle \mathbb{Z}, + \rangle$ is the map defined by $\phi(n) = n + 1$ for $n \in \mathbb{Z}$. Explain why $\phi$ is *not* an isomorphism.

**c.** (*6 pts*) Suppose $\psi\colon \langle \mathbb{Z}, + \rangle \to \langle \mathbb{Z}, + \rangle$ is the map defined by $\psi(n) = 2n$ for $n \in \mathbb{Z}$. Explain why $\psi$ is *not* an isomorphism.

**d.** (*6 pts*) Let $M_n(\mathbb{R})$ be the set of all $n \times n$ matrices with real entries. Explain why $M_n(\mathbb{R})$ is *not* a group under matrix multiplication.

**Survey Questions:** *(2 extra credit points)*

Name a question or topic that could have been on this test, but wasn't.

How do you think you did?

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**SPACE FOR SCRAP WORK**