IN-CLASS TEST II

Math 430
October 22, 2004

Name: ______________________________
By writing my name I swear by the honor code.

Read all of the following information before starting the exam:

- Your work will be graded for clarity as well as for mathematical accuracy. Make sure that your logic is clear and that I can tell what you are proving, and how. Provide reasons for steps whenever possible.
- Don't get hung up on any one problem. If you get stuck, move on and come back to the problem later.
- By writing your name above, you agree to the JMU honor code. In particular, this means that you may not use any notes or crib sheets during this exam, that all work must be your own, and that you may not obtain advance information revealing the problems on this exam.
- This test has 4 multi-part problems and is worth 100 points, plus some extra credit at the end. Make sure that you have all of the pages!
- Good luck!

1. (*15 pts*) **True/False.** Determine whether each statement is true (T) or false (F).

(**a**) **T** **F** Every element of a group generates a cyclic subgroup of the group.

(**b**) **T** **F** Every subgroup is a left coset.

(**c**) **T** **F** Every left coset is a subgroup.

(**d**) **T** **F** If G is an infinite cyclic group, then G is isomorphic to the group $\langle \mathbb{Z}, + \rangle$.

(**e**) **T** **F** No group of order 12 can have a subgroup of order 5.

2. (*25 pts*) **Examples.** Give an example of each of the following, if possible. You do not have to prove that your example satisfies the conditions listed. If such an example does not exist, write "NOT POSSIBLE."

(**a**) An abelian group that is not cyclic.

(**b**) A cyclic group that is not abelian.

(**c**) A group with exactly two nontrivial proper subgroups.

(**d**) A subset of a group that is not a subgroup.

(**e**) A nontrivial abelian subgroup of a nonabelian group.

3. (*30 pts*) **Calculations.** Showing work might get you partial credit. Circle your final answers. For parts (d), (e), and (f) you will need the footnote at the bottom of the page.

(a) List the elements of the subgroup generated by the subset $\{4, 6\}$ of $\mathbb{Z}_{12}$.

(b) If $\sigma = (1\,2\,5\,4)(2\,6\,1) \in S_6$, find $|\langle\sigma\rangle|$.

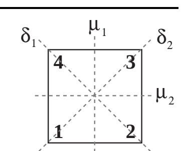
(c) Find the number of generators of the cyclic group $\mathbb{Z}_{35}$. (In other words, how many elements $k \in \mathbb{Z}_{35}$ will generate the entire group?)

(d) $\mu_1\rho_2\delta_2$ is one of the eight elements of D_4. Which one?

(e) List the orbits of the element $\delta_1 \in D_4$ (thought of as a permutation in S_4).

(f) List all of the elements of D_4 that are also elements of the alternating group A_4.

Recall that D_4, the group of symmetries of the square, has eight elements: ρ_0 = identity, ρ_1 = counterclockwise rotation by 90 degrees, $\rho_2 = \rho_1^2$, $\rho_3 = \rho_1^3$, and four reflections μ_1, μ_2, δ_1, and δ_2 whose axes of reflection are shown in the figure.



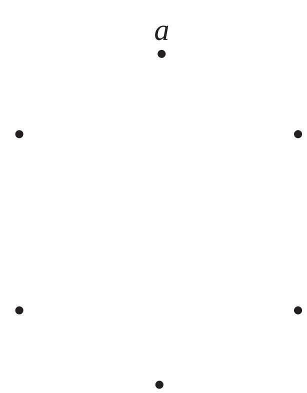
4. (*30 pts*) **Group Table.** Suppose G is the group whose multiplication table is given below. Use the table to answer the following questions.

(**a**) Make a subgroup diagram for G.

	a	b	c	d	f	g
a	a	b	c	d	f	g
b	b	a	d	c	g	f
c	c	f	g	b	d	a
d	d	g	f	a	c	b
f	f	c	b	g	a	d
g	g	d	a	f	b	c

(**b**) Find all of the left cosets of the cyclic subgroup $\langle c \rangle$ of G.

(**c**) Make a Cayley graph for G using the generating set $\{b, c\}$, using the six dots below.



Survey Questions: *(2 extra credit points)*

Name a question or topic that could have been on this test, but wasn't.

How do you think you did?

SPACE FOR SCRAP WORK