# IN-CLASS TEST III

Math 430
November 19, 2004

**Name:** ________________________________

By writing my name I swear by the honor code.

**Read all of the following information before starting the exam:**

- Your work will be graded for clarity as well as for mathematical accuracy. Make sure that your logic is clear and that I can tell what you are proving, and how. Provide reasons for steps whenever possible.
- Don't get hung up on any one problem. If you get stuck, move on and come back to the problem later.
- By writing your name above, you agree to the JMU honor code. In particular, this means that you may not use any notes or crib sheets during this exam, that all work must be your own, and that you may not obtain advance information revealing the problems on this exam.
- This test has xxx multi-part problems and is worth 100 points, plus some extra credit at the end. Make sure that you have all of the pages!
- Good luck!

**1.** (*15 pts*) **True/False.** Determine whether each statement is true (T) or false (F).

(a) **T** **F** If $G_1$ and $G_2$ are any groups, then $G_1 \times G_2$ is always isomorphic to $G_2 \times G_1$.

(b) **T** **F** $\mathbb{Z}_{18} \times \mathbb{Z}_{35}$ is a cyclic group.

(c) **T** **F** The characteristic of the ring $7\mathbb{Z}$ is 7.

(d) **T** **F** If $D$ is an integral domain, then every field containing $D$ contains a subfield which is a field of quotients of $D$.

(e) **T** **F** $\mathbb{C}$ is a field of quotients of $\mathbb{R}$.

**2.** (*25 pts*) **Examples.** Give an example of each of the following, if possible. You do not have to prove that your example satisfies the conditions listed. If such an example does not exist, write "NOT POSSIBLE."

(a) A finite commutative ring with unity that is not an integral domain.

(b) A non-commutative ring with unity in which every nonzero element is a unit.

(c) A function $\phi\colon \mathbb{Z}_3 \to \mathbb{Z}_4$ that is not a group homomorphism.

(d) A group homomorphism $\phi\colon \mathbb{Z} \to S_6$ whose kernel is $4\mathbb{Z}$.

(e) A nontrivial ring homomorphism $\phi\colon \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$.

**3.** (*30 pts*) **Proofs.** Your arguments will be graded for clarity, brevity, and completeness, as well as mathematical accuracy. You may find it useful to collect your thoughts on the scrap page and then carefully write your final arguments here.

(**a**) Prove that any group homomorphism $\phi\colon G \to G'$ where $|G|$ is a prime must be either the trivial homomorphism or a one-to-one map.

(**b**) Prove that the rings $\mathbb{Z}$ and $3\mathbb{Z}$ are not isomorphic.

**4.** (*30 pts*) **Calculations.** Showing work might get you partial credit. Circle your final answers.

**(a)** Find the order of $(8, 4, 10)$ in the group $\mathbb{Z}_{12} \times \mathbb{Z}_{60} \times \mathbb{Z}_{24}$.

**(b)** Find the number of abelian groups of order 400, up to isomorphism.

**(c)** Consider the group homomorphism $\phi\colon \mathbb{Z}_{10} \to \mathbb{Z}_{20}$ defined by $\phi(1) = 8$. How many cosets does the subgroup $\ker(\phi)$ have?

**(d)** List all of the units in the ring $\mathbb{Z}_6 \times \mathbb{Z}$.

**(e)** List all of the zero divisors in the ring $\mathbb{Z}_2 \times \mathbb{Z}_4$.

**Survey Questions:** *(2 extra credit points)*

Name a question or topic that could have been on this test, but wasn't.

How do you think you did?

---

**SPACE FOR SCRAP WORK**