Puzzling over Sudoku

Explaining — and expanding on — the pop-culture phenomenon By Laura Taalman, Ph.D.

America has been taken over by little 9 by 9 grids full of numbers. Sudoku puzzles are now a regular feature in almost every newspaper, and bookstores devote entire sections to Sudoku books. But, we're late to the party; Sudoku has been popular since the '80s in Japan after its first appearance in print in an American puzzle magazine in 1979. The roots of Sudoku are even older, dating back to Latin squares and magic squares, which have been studied for hundreds of years.

In case you've missed the phenomenon, here is how Sudoku puzzles work: each puzzle is a 9 by 9 grid with some numbers filled in, and your job is to fill in the rest of the grid so that every row, column and 3 by 3 block contains each of the whole numbers from 1 to 9 exactly once. Sudoku puzzles are always constructed so that there is only one possible way to fill in the grid.

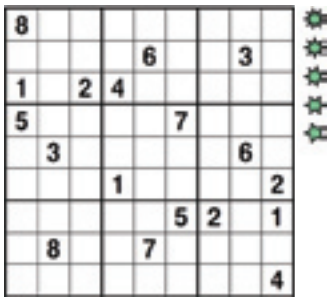
Many puzzle authors also impose the condition that the set of cells in which the initial clues appear has 180-degree rotational symmetry. This means that if you rotate the puzzle 180 degrees, the cells with clues appear in the same locations. This symmetry convention is also followed in the black squares of most crossword puzzles. Although this symmetry does not add anything to the experience of playing the puzzle, it does make the puzzles more pleasing to the eye.

One of the most basic questions that can be asked about Sudoku puzzles is an unsolved problem: how many clues are needed to guarantee a unique solution? Mathematicians and computer scientists have conjectured that at least 17 clues are always needed. Although there are many known 17-clue puzzles and no known 16-clue Sudoku puzzles, it is still an open problem to *prove* that no 16-clue puzzles exist. When the rotational symmetry condition is imposed, the conjectured number jumps to 18; no 17-clue rotationally symmetric Sudoku puzzles have yet been found. You won't usually see 18-clue puzzles in Sudoku books, because they can be quite difficult. If you're up to a challenge, try the puzzle below left. The unique solution to this puzzle is below right; notice that each of the numbers 1 through 9 appears exactly once in each row, column and block of the solution.

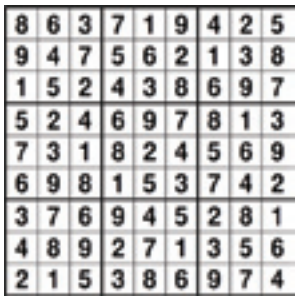
Although puzzles with fewer clues tend to be more difficult than puzzles with more clues, the number of clues alone does not determine the difficulty of a Sudoku puzzle. Because difficulty level is subjective, it is hard to accurately rate Sudoku puzzles. Different puzzle makers will use different methods to rate their puzzles, usually using computer programs written for that purpose. Some puzzle rating programs apply well-known human solving techniques to a puzzle and keep track of the complexity of techniques required to solve the puzzle. This method of rating works well if the people later playing the puzzle tend to use the same solving techniques used by the computer program. If you've ever played a supposedly easy-rated puzzle and found it very challenging (or vice-versa), it may be that you do not usually use the same solving techniques used by the rating program.

A more objective way of rating puzzles, developed by JMU alumnus Philip Riley ('02M), is to have a computer play a particular puzzle thousands of times in thousands of different ways and keep a running average of the time and number of steps required to solve the puzzle. This is in fact how the puzzles in this article were rated. However, since the puzzles are not all of the same type, you still might find some puzzles inherently harder than others. One-star puzzles are meant to be very easy, and five-star puzzles should keep you awake at night.

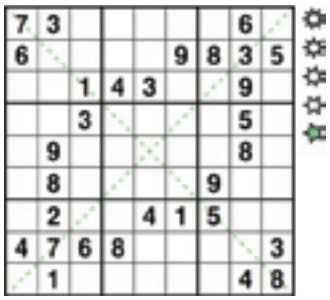
But perhaps you are a Sudoku expert already? Are you bored with solving the same type of puzzle over and over again? Do you need a new challenge? There are many ways of adding new rules and conditions to Sudoku that result in puzzles that require new solving strategies. One very simple way to do this is to require that the diagonals of the puzzle also contain the numbers 1 through 9 exactly once, as in "Sudoku X" below. This does not necessarily make the puzzle easier, since the new information from the diagonal regions means that the puzzle maker could potentially use fewer initial clues. This easy Sudoku X puzzle has 30 clues, but there exist Sudoku X puzzles with as few as 13 clues. Another variation on Sudoku is shown in "Pyramids" below right, in which the four pyramid regions must also contain the numbers 1 through 9 exactly once.



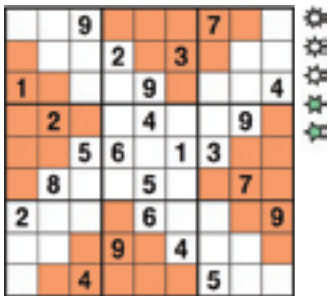
18-clue Sudoku: *Each row, column, and block must contain 1 to 9 exactly once.*



The solution to the puzzle.



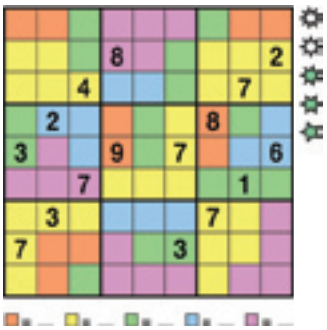
Sudoku X: *Each row, column, block and main diagonal must contain 1 to 9 exactly once.*



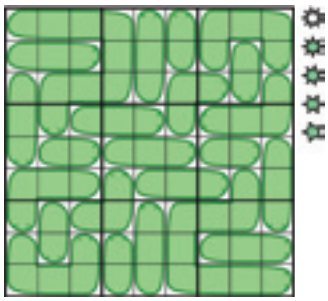
Pyramids: *Each row, column, block and pyramid region must contain 1 to 9 exactly once.*

PUZZLES COURTESY OF LAURA TAALMAN

Newspapers that run Sudoku puzzles often reassure their readers that although the puzzles involve numbers, they do not require any mathematics. What they really mean is that Sudoku does not require any *arithmetic.* We do not need to worry about the numerical properties of the numbers in the puzzle. However, the heart of mathematics is not numbers and arithmetic, but rather reasoning and logic. In this sense, Sudoku puzzles are entirely about mathematics. Of course, we could always put some arithmetic into a Sudoku puzzle just for fun, as in the "Mystery Sums" and "Worms" puzzles, right. Notice that the Mystery Sums puzzle offers only 16 clues — fewer clues than any known standard Sudoku puzzle — because it has more conditions than just rows, columns and blocks. In fact, at least 10 initial clues in this puzzle are not actually needed to guarantee a unique solution. They are only included here so that the puzzle is not prohibitively difficult. In other words, this Mystery Sums puzzle could have been a six-clue puzzle.



Mystery Sums: *Each row, column and block must contain 1 to 9 exactly once. Regions of the same color add to the same sum, which you must determine.*



Worms: *Each row, column and block must contain 1 to 9 exactly once. The entries in each worm decrease from head to tail (but not necessarily sequentially). For example, a worm could contain 2, 3, 6, 8, in that order, from head to tail.*

Even worse, the Worms puzzle has no initial clues! The worm conditions are so restrictive that there is only one possible solution to the puzzle, even though no initial clues are given.

So where do Sudoku puzzles come from? The puzzles in most Sudoku books sold in the United States are generated by computer, sometimes by starting with a completed grid and removing entries one by one, and sometimes by starting with a blank grid and adding entries one by one, at each step checking to see if the puzzle has a unique solution. There are Japanese puzzle companies that make all of their wonderful puzzles entirely by hand. The puzzles in this article were made by myself and Philip Riley, together known as Brainfreeze Puzzles. We make "cyborg" puzzles, meaning that we use computers for the tedious parts of the process but design and build each puzzle individually by hand. The "James Madison Sudoku" puzzle below was made especially for *Madison* magazine readers. M

About the Author: *Mathematics professor Laura Taalman earned her Ph.D in mathematics from Duke University and completed undergraduate work at the University of Chicago. Her research includes singular algebraic geometry, knot theory and the mathematics of Sudoku and other puzzles. She has written a textbook that combines calculus, pre-calculus, and algebra into one course, and she is one of the organizers of the Shenandoah Undergraduate Mathematics and Statistics Conference at JMU. The Mathematical Association of America has honored her with the Trevor Evans Award and the Alder Award for Distinguished Teaching.*

JAMES MADISON

SUDOKU CONTEST

WIN A COPY OF LAURA TAALMAN'S NEW BOOK: *Color Sudoku*

Directions:

Each row, column and jigsaw region must contain exactly the letters that appear in "JAMES MADISON." Notice that the letters A, M and S will each appear twice in every row, column and region.

How to enter:

Submit your completed puzzle by Oct. 15 to be eligible for a drawing Homecoming Weekend. Ten readers will win Brainfreeze Puzzles' new book *Color Sudoku.* Mail entries to: *Madison* magazine, MSC 3610, 220 University Blvd., JMU, Harrisonburg, VA 22807. Be sure to include your name, class year, mailing address and major.