Research Statement
Roger Thelwell

My current research area is in nonlinear waves. This is a relatively new area for me, which I entered with my postdoctoral appointment. It is an incredibly rich area, with applications including deep and shallow water waves, optics, Bose-Einstein Condensates and plasmas. I am collaborating with specialists in the fields of physical experimentation, perturbation theory, scientific computation, algebraic geometry and mathematical analysis.

My thesis work was in the field of inverse problems. This is a rapidly growing area, with numerous research topics. Model fitting, medical and seismic imaging and non-destructive testing are just a few applications. The majority of open inverse problems are inherently ill posed. This difficulty is further compounded by the observation that each research problem typically requires its own unique approaches, skills and techniques. I have worked on several inverse problems, all of which admit the possibility of future work. My work uses functional analysis, PDE theory, numerical analysis as well as numerical experimentation.

Nonlinear Waves

Waves play an important role in the open ocean and coastal regions. My current research is motivated by patterns in three-dimensional surface water waves. Observations, both in nature and in the lab, suggest that waves of permanent form might exist. If this can be established mathematically, a practical theory for inviscid three-dimensional water waves is possible. The uniform plane wave, first formalized by Stokes' as a solution of the linearized Euler equation, is one of an infinite family of forms that I am considering as potential building blocks for this theory. Although the famous paper of Benjamin and Feir showed that these waves are modulationally unstable in deep water, perhaps other members of the family are stable. The nonlinear Schrödinger (NLS) and the Korteweg-deVries (KdV) equations have solutions that represent amplitudes of two-dimensional surface water waves. The NLS equation models waves in deep water, and the KdV equation in used to model shallow water waves.

I am intrigued by the possibility that stable, fully three-dimensional solutions might be found. I recently finished a detailed stability study of a large class of NLS solutions, in the hope that some might prove stable, and thus could serve as building blocks for a theory of waves in deep water. I now believe that there are no solutions which are stable under two-dimensional perturbations. I am currently exploring two-dimensional solutions in the shallow water KdV setting, a far more likely regime for persistent forms. The stability methods I use are spectrally based, highly efficient and applicable to a wide class of problems. A natural extension would be to apply these methods to other systems, including the Kadomtsev-Petviashvili (KP) equation.

I am also interested in the feasibility of using surface measurements to recover bottom topography. The Zakharov formulation of the free surface water wave in finite depth might make this feasible - since the canonical variables are naturally surface quantities, and therefore measurable. Cast in these variables, however, the inversion appears difficult. Perhaps Dirichlet-to-Neumann methods can be used to propagate this surface information vertically.

Porous Media

My thesis research involves an integral based inverse method as a means to recover the coefficients $C(h)$ and $K(h)$ in a nonlinear parabolic partial differential equation of the form

$$C(h)\partial_t h = \partial_x(K(h)\partial_x(h-1)).$$

In porous media applications, this equation is referred to as the Richards Equation, and is widely used to model fluid flow in porous media. Meaningful solutions require an accurate description of soil characteristics, reflected in the coefficients $C(h)$ and $K(h)$. I have been working with my thesis advisor, Paul DuChateau, on recovery of these coefficients, which we consider as inputs, from observable measurements, or outputs, of a physical experiment. Our indirect method, based on an integral identity, uses an adjoint approach. By careful choice of the adjoint problem and an appropriate selection of control in the physical experiment, explicit properties of the input-to-output map become accessible. Additionally, the integral identity method is quite stable under noise. Carleman estimates and Output Least Squares (OLS) are typically used to attack this problem, but our method provides immediate insight of the input-to-output map. A technical paper describing our method for a reduced equation was one of the most downloaded articles in 2004 from the journal Inverse Problems. A numerical counterpart article is in preparation, as applied to the Richards equation.

We assume homogeneous media and neglect hysteresis. The current theory might be extended to provide insight and coefficient recovery in anisotropic flow situations. Although more difficult, spatially dependent coefficient identification is another avenue of research. Perhaps a suitable integral identity type approach might help to explain this difficulty. Physical experiments commonly reveal hysteretic coefficients. From scanning data, our methods should be able to recover these features. Another extension of this work is to consider multi-phase flow, as the current identity method appears to admit this possibility.

Biological models

Joe von Fischer, a biologist at Colorado State, and I are trying to understand the process of methane production and consumption in soil. Methane is an important greenhouse gas, and this research should help provide insight into the physics of the system. We are currently developing and tuning our model, using laboratory data to guide us. The next phase will be to consider convection terms and spatially dependent parameters, as might be observed in wetlands. Professor von Fischer recently received NSF funding that will allow me to increase my level of involvement in this project.

Atmospheric Dynamics

As a sailor, I have interest in the models used for hurricane analysis, estimation and prediction. I worked for the Atmospheric Science Department at Colorado State in the effort to understand the dynamics of cyclonic flow, and encountered a wonderful inverse problem. Given a prescribed pressure gradient field, or a time series of these fields, can a matching velocity field be recovered? If so, how ill-posed is the recovery? The accessibility of pressure information from highly resolved satellite imaging, coupled with the Navier-Stokes equation, would seem to allow this type of recovery. In fact, this is still an open problem.

Stochastic PDE's

I know little about stochastic PDEs, but I find their broad applicability and intuitive nature very appealing. I would like to learn more about this field, specifically as applied to inverse problems.