The Restricted Hill's Problem (and the Power Series Method)

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Outline

Abstract

We present an approach to recast non-polynomial IVODEs into polynomial differential systems which exploit auxiliary variables, and provide several examples using the Power Series Method (PSM). We review an *a priori* error bound for the solution to IVODES of polynomial form. We apply this approach to Hill's Restricted Three Body Problem of celestial mechanics to demonstrate that PSM is effectively symplectic. If we have time, we will offer some novel conservation expressions for this system, and a general approach to generate others.

Act I:
Power series

$y' = \alpha y^2$

We start with a toy problem...

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An analytic solution to this one is easy:

$$y(t) = -\frac{y_0}{\alpha y_0 t - 1}$$

$y' = \alpha y^2$: series solution

What happens if we try (formal, for now) series?

$y' = \alpha y^2$: series solution

What happens if we try (formal, for now) series?
Let

$$y(t) = \sum_{k=0}^{\infty} y_k t^k$$

then

$$\sum_{k=0}^{\infty} (k+1) y_{k+1} t^k = \alpha \sum_{k=0}^{\infty} \left(\sum_{i,j \geq 0}^{i+j=k} y_i y_j \right) t^k$$

so we equate coefficients to get

$$y_{k+1} = \frac{\alpha}{k+1} \sum_{i+j=k} y_i y_j$$

$y' = \alpha y^2$: Maple

Or, with MAPLE:

```
> ODE1 := diff(y(t),t) = alpha*y(t)^2;
> IC := y(0) = y0;
> y1 := dsolve({ODE1,IC},y(t))
```

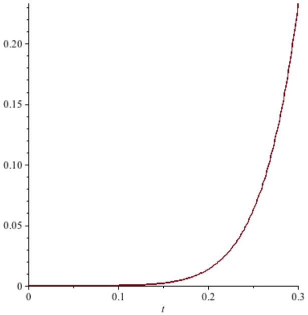
and

```
> Y1 := dsolve({ODE1,IC},y(t),series);
       Y1 := 1+2*alpha*t*y0+3*alpha^2*y0^2*t^2+...
```

$y' = \alpha y^2$: Error

What about the error?

```
> alpha := 1; y0 := 2; plot(abs(Y1-y1),t=0..0.5);
```



Can we quantify this?

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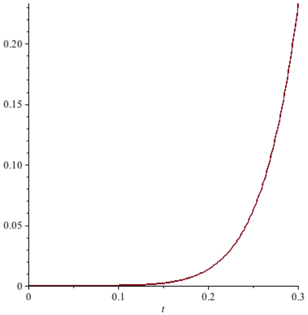

Can we quantify this?
Let's take a little diversion...

$y' = \alpha y^m$: important aside

The solution to the constant coefficient nonlinear IVODE

$$y' = \alpha y^m \qquad y(0) = y_0$$

is messy:

$$y(t) = \left((\alpha - \alpha m)t + y_0^{1-m}\right)^{-(m-1)^{-1}}.$$

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But the ratio $\frac{y'}{y}$ isn't!

$$\frac{y'}{y} = \frac{\alpha}{(\alpha - \alpha m)t + y_0^{1-m}}$$

and we see that

$$y'(t) = \underbrace{\frac{\alpha}{(\alpha - \alpha m)t + y_0^{1-m}}}_{K(t)} y(t),$$

is a non-constant coefficient LINEAR ode.

$y' = \alpha y^m$: important aside

So

$$y'(t) = \underbrace{\frac{\alpha}{(\alpha - \alpha t m)t + y_0^{1-m}}}_{K(t)} y(t),$$

has solution

$$y(t) = y_0 exp\left(\int_0^t K(\tau) d\tau\right)$$

or, via series,

$$Y_{k+1} = \frac{\alpha(1 + (m-1)k)}{y_0^{1-m}(k+1)} Y_k$$

$y' = \alpha y^m$: important aside

From

$$Y_{k+1} = \frac{\alpha(1 + (m-1)k)}{y_0^{1-m}(k+1)} Y_k$$

and for $m \geq 2$,

$$Y_{k+1} \leq \alpha(m-1)|y_0|^{m-1} Y_k \leq \alpha C_\infty Y_k.$$

This leads directly to a geometric series bounding $y(t)$:

$$y(t) \leq \frac{|y_0 \alpha|}{1 - C_\infty t} = |y_0 \alpha| \sum_{k=0} (C_\infty t)^k$$

Now for the bound...

$y' = \alpha y^2$: back to error

From

$$y(t) \le \frac{|y_0\alpha|}{1 - C_\infty t} = |y_0\alpha| \sum_{k=0} (C_\infty t)^k$$

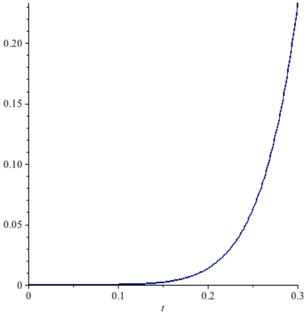
we see that the absolute error is

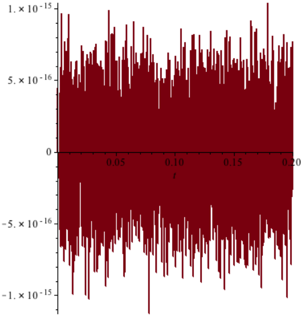
$$|Y1 - y1| \le |y_0\alpha| \sum_{k=n+1}^{\infty} C_\infty |t|^k \le \frac{|y_0\alpha| C_\infty^{n+1}}{1 - C_\infty |t|}$$

where $C_\infty = |y_0\alpha|$.
An ERROR bound!

$y' = \alpha y^2$: error plots

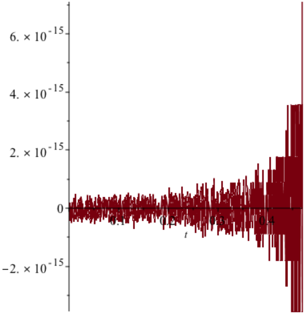
```
> ee :=  abs(Y1-y1);
> m := 2; Cinf := y0*alpha;
> EE := N -> abs(y0)*(Cinf*t)^(N+1)/( 1 - Cinf*abs(t));
> plot({ee,EE(5)},t=0..0.3);
> plot({ee-EE(5)},t=0..0.2);
```





$y' = 2y^2, \quad y(0) = 1$: radius of convergence?

```
>plot({ee-EE(5)},t=0..0.48);
```



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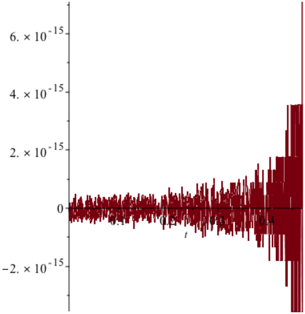

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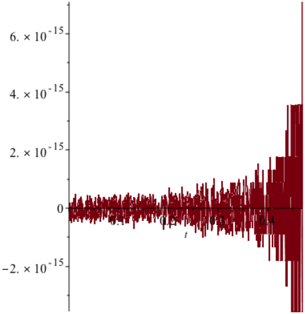


Hmmmm....

$$y(t) = -\frac{y_0}{\alpha y_0 t - 1} \qquad \text{with } \alpha = 2$$

$y' = \alpha y^2$: radius of convergence

But what if we only have this form?

```
> Y20 := dsolve({ODE1,IC},y(t),series);
       Y20 := 1+2*alpha*t*y0+3*alpha^2*y0^2*t^2+...
```

$y' = \alpha y^2$: radius of convergence

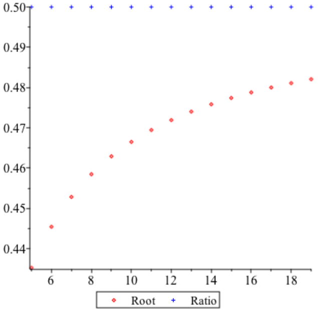
But what if we only have this form?

```
> Y20 := dsolve({ODE1,IC},y(t),series);
      Y20 := 1+2*alpha*t*y0+3*alpha^2*y0^2*t^2+...
```

Approximate using the root or ratio test. Or calculate the Padé approximant.

$y' = 2y^2, y(0) = 1$: radius of convergence?

```
> Order := 20: alpha := 2; y0 := 1;
> Y20 := rhs(dsolve({ODE1,IC},y(t),series)):
> Ycoeff := [seq(coeff(convert(Y20,polynom),t,i),i=1..20)];
> RatioT := i -> abs(a[i+1]/a[i]): RootT := i -> abs(a[i])^(1/
> RootEst := [seq(1/RootT(i),i=1..Order)]: RatioEst := [seq(1/
> plot ....
```



Act II
Hill's Lunar 3 body

Timeline

The Sun, the Earth, and the Moon

- 1000 B.C.E Babylonians
- 200 B.C.E Greeks
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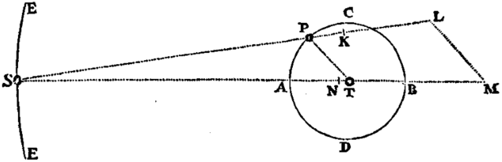

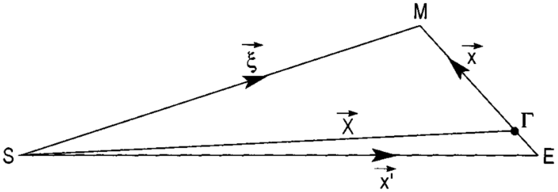

Timeline

The Sun, the Earth, and the Moon

- 1000 B.C.E Babylonians
- 200 B.C.E Greeks
- (1543-1609) Copernicus to Kepler
- 1687 Newton (I.66)
- 1740-55 Clairaut, Euler, and d'Alembert
- 1767 Euler
- 1860-67 Delaunay
- 1877-1878 Hill

Hill's Lunar equations

Hill's 1877 "On the Part of the Motion of the Lunar Perigee which is a Function of the Mean Motions of the Sun and the Moon"

Key features:

- earth-centric coordinate frame
- sun massive, infinitely far away.
- moon as point mass in rotating coordinate, one axis towards sun.
- looked for a periodic orbit of a perturbed system
- not a systematic perturbation theory, but thoughtful expansion of variables

For much more detail, see survey by:

Martin C. Gutzwiller, Moon-Earth-Sun: The oldest three-body problem, Rev. Mod. Phys. 1998 [4]

Hill's Lunar equations

From [Waldvogel, 1997] in geocentric cartesian (x, y):

$$\ddot{x} - 2\dot{y} = 3x - xr^{-3} \tag{1}$$

$$\ddot{y} + 2\dot{x} = \quad + yr^{-3} \tag{2}$$

Or, in conjugate momenta (q, p), with
$q_1 = x, q_2 = y, p_1 = \dot{q}_1 - q_2,$ and $p_2 = \dot{q}_2 + q_1,$

$$\dot{q}_1 = p_1 + q_2 \tag{3}$$

$$\dot{q}_2 = p_2 - q_1 \tag{4}$$

$$\dot{p}_1 = \ p_2 + 2q_1 - q_1 r^{-3} \tag{5}$$

$$\dot{p}_2 = -p_1 - q_2 - q_2 r^{-3}, \tag{6}$$

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$$\dot{p}_2 = -p_1 - q_2 - q_2 r^{-3}, \quad (6)$$

with a conserved quantity $h = \frac{1}{2}\left(\dot{x}^2 + \dot{y}^2\right) - \frac{3}{2}x^2 - \frac{1}{r}, \quad r = \sqrt{x^2 + y^2}$ or
$H(q, p) = \frac{1}{2}(p_1^2 + p_2^2) + p_1 q_2 - p_2 q_1 - q_1^2 + \frac{1}{2}q_2^2 - \frac{1}{r}, \quad r = \sqrt{q_1^2 + q_2^2}.$

Symplectic integrator?

Powerful symplectic tools are available, such as:

- Martin Hairer's `GNI` package (link here)
- `SymInt` package (link here)
- Velocity Verlet (one link here)

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Unfortunately, these general integrators are for canonical second order system. They require that

$$\vec{U}'' = F(t, \vec{U}),$$

or

$$\vec{U}' = \vec{V} \tag{7}$$

$$\vec{V}' = F(t, \vec{U}), \tag{8}$$

where U is position and V is velocity.

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$$\vec{V}' = F(t, \vec{U}), \tag{8}$$

where U is position and V is velocity.

We have a velocity-dependent force.

Symplectic integrator

What to do? Several authors have constructed special formulations to handle non-symmetry in Hills three body, which include:

- [Waldvogel, 1997] ‘Symplectic Integrator’s for Hill’s Lunar Problem‘
- [Quinn, 2010] ‘A Symplectic Integrator for Hill’s Equations’

Both of these require gymnastics: finding a canonical transform to build new Hamiltonian $K(u, v) = K_1(u, v) + K_2(u)$, such that K_1 and K_2 are both integrable.

Hill's Lunar equations: Symplectic integrator?

Power series?

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Power series methods may provide *effectively symplectic* integration of conservative systems through a (numerically) faithful algorithm.

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Power series?

Power series methods may provide *effectively symplectic* integration of conservative systems through a (numerically) faithful algorithm.

```
function dYdt = fhill3_xy(t,Y)

recipr3 = 1/(Y(1)^2 + Y(3)^2)^(3/2);

dYdt = [ Y(2);
    2*Y(4) + 3*Y(1) - Y(1)*recipr3;
    Y(4);
    -2*Y(2) - Y(3)*recipr3];
end
```

Hill's Lunar equations: PSM (x,y)

```
>> analyze(fhill3_xy,0,[1;0;0;1])
    'Definition          Series recur *=CP                     '
    'u1 = y1 * y1        u1 = y1 * y1                          '
    'u2 = y3 * y3        u2 = y3 * y3                          '
    'u3 = u1 + u2                                              '
    'u4 = u3^1.5         u3 * u4' = 1.5 u4 * u3' solve for u4(k)'
    'u5 = 1 / u4         1 = u5 * u4 solve for u5(k)           '
    'u6 = 2 * y4                                               '
    'u7 = 3 * y1                                               '
    'u8 = u6 + u7                                              '
    'u9 = y1 * u5        u9 = y1 * u5                          '
    'u10 = u8 - u9                                             '
    'u11 = -2 * y2                                             '
    'u12 = y3 * u5       u12 = y3 * u5                         '
    'u13 = u11 - u12                                           '
    '                    y1' = y2                              '
    '                    y2' = u10                             '
    '                    y3' = y4                              '
    '                    y4' = u13                             '
```

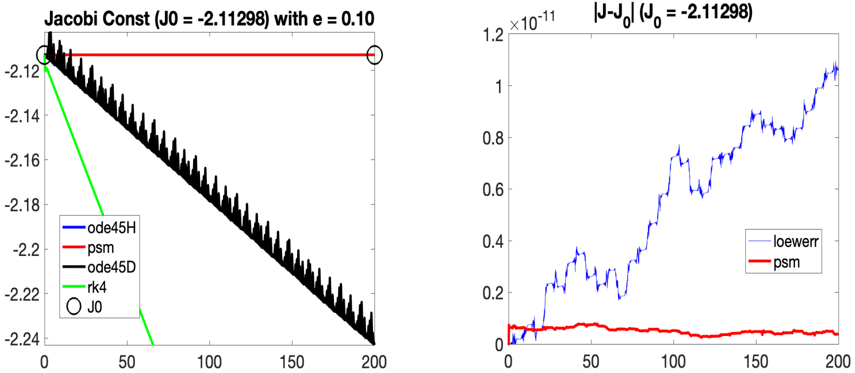
Hill's Lunar equations: PSM (q,p)

```
>> analyze(fhill3_pq,0,[1;0;0;1])

    'Definition          Series recur *=CP                    '
    'u1 = y1 * y1        u1 = y1 * y1                         '
    'u2 = y2 * y2        u2 = y2 * y2                         '
    'u3 = u1 + u2                                             '
    'u4 = u3^1.5         u3 * u4' = 1.5 u4 * u3' solve for u4(k)'
    'u5 = 1 / u4         1 = u5 * u4 solve for u5(k)          '
    'u6 = y3 + y2                                             '
    'u7 = y4 - y1                                             '
    'u8 = 2 * y1                                              '
    'u9 = y4 + u8                                             '
    'u10 = y1 * u5       u10 = y1 * u5                        '
    'u11 = u9 - u10                                           '
    'u12 = -2 * y4                                            '
    'u13 = u12 - y2                                           '
    'u14 = y2 * u5       u14 = y2 * u5                        '
    'u15 = u13 - u14                                          '
    '                    y1' = u6                             '
    '                    y2' = u7                             '
    '                    y3' = u11                            '
    '                    y4' = u15                            '
```

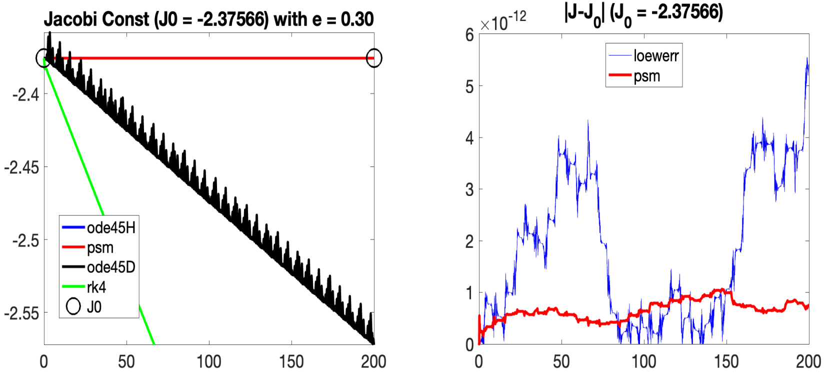
Hill's Lunar equations: 3 body

$e = 0.1$



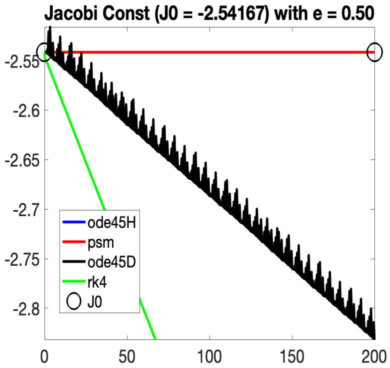
Hill's Lunar equations: 3 body

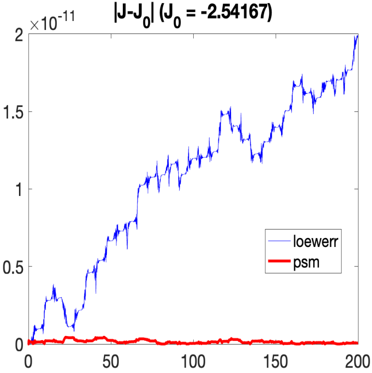
$e = 0.3$



Hill's Lunar equations: 3 body

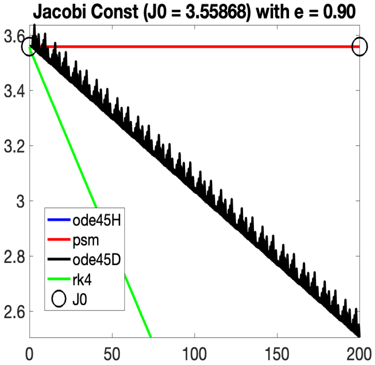
$e = 0.5$

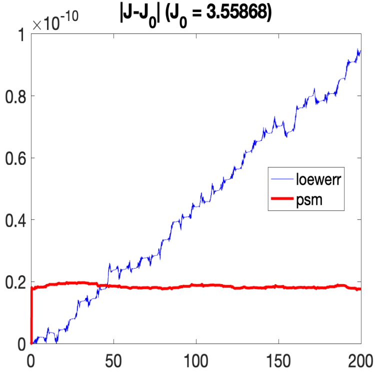




Hill's Lunar equations: 3 body

$e = 0.9$





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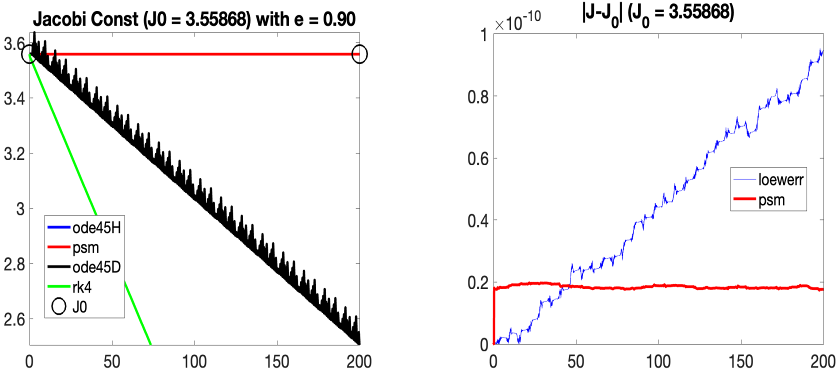
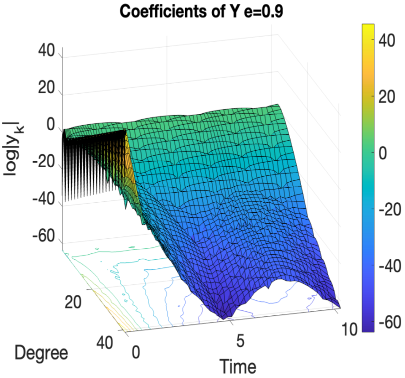


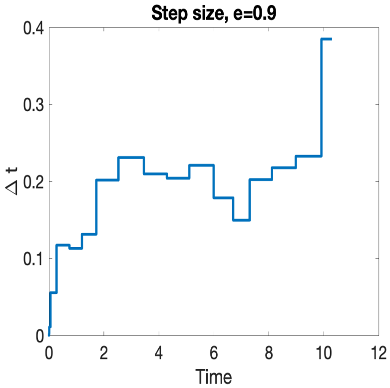
Hmmmm..... Can we do better?

Hill's Lunar equations: 3 body

What do the coefficients look like?

$$e = 0.9$$

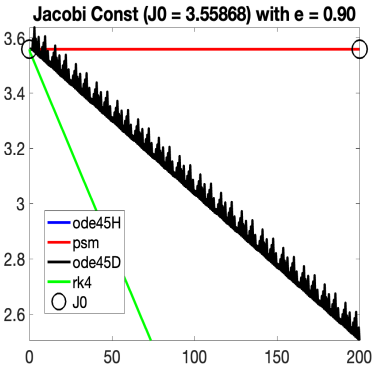


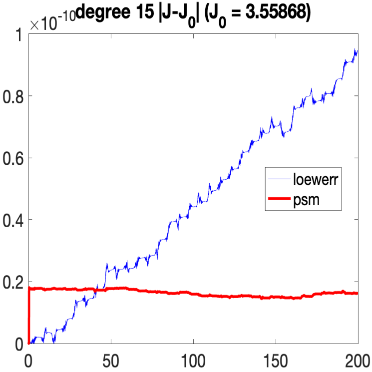


Hill's Lunar equations: 3 body

Refine step?

Back to $e = 0.9$ with refined step.





Hill's Lunar equations: 3 body

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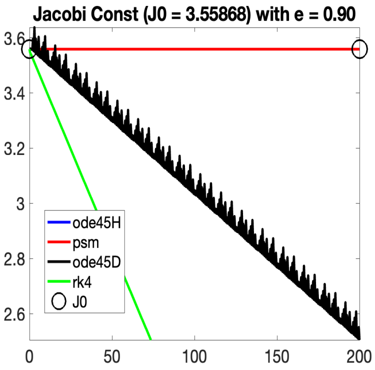
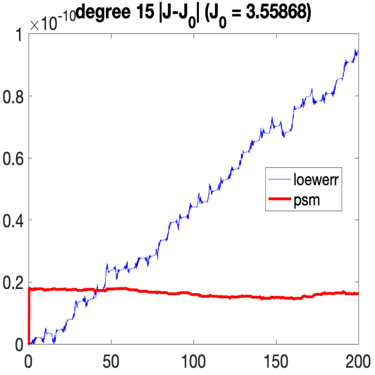


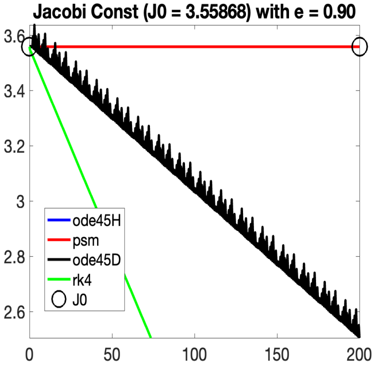


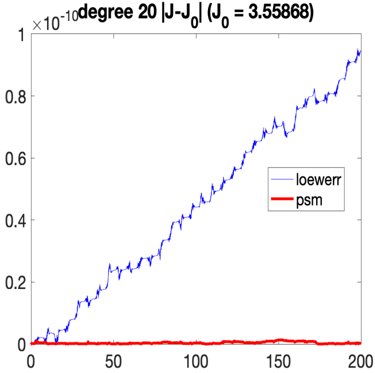
Hmmmmm.......

Hill's Lunar equations: 3 body

What if we raise degree?

Back to $e = 0.9$ with higher degree.





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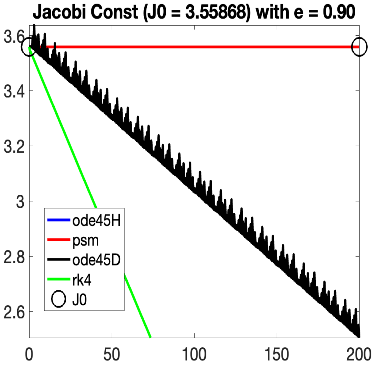

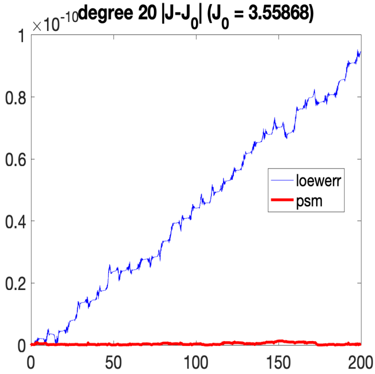

Conclusions

Our PSM methods rely on the use of auxiliary variables to build a system of polynomial IVODEs. Once the system is polynomial, series methods allow remarkably direct analysis. We made a small study of Hill's 3 body lunar problem and demonstrated:

- Effectively symplectic
- Easy to apply
- Easy error BOUND
- Transparent information

These techniques should apply to a broad range of highly nonlinear ODE. And something we didn't talk about - PSM techniques can be used to identify numerically conserved quantities for validation.

Thank you

Thanks!

Questions?

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Conserved quantities in a polynomial system

Start with

$$\begin{aligned} x_1' &= x_2 \\ x_2' &= 2x_4 + 3x_1 - x_1 x_5^3 \\ x_3' &= x_4 \\ x_4' &= -2x_2 - x_3 x_5^3 \\ x_5' &= -x_1 x_2 x_5^3 - x_3 x_4 x_5^3 \\ r' &= x_1 x_2 x_5 + x_3 x_4 x_5 \end{aligned}$$

Then

$$x_2 x_2' = 2x_2 x_4 + 3x_1 x_2 - x_1 x_2 x_5^3 \text{ and } x_4 x_4' = -2x_2 x_4 - x_3 x_4 x_5^3,$$

so

$$x_5' - x_2 x_2' - x_4 x_4' = -3x_1 x_2 = -3x_1 x_1'.$$

And we have a conserved (Jacobi) constant:

$$2h = 2x_5 - x_2^2 - x_4^2 + 3x_1^2.$$

Act III
The 4 Body Problem

Hill's Lunar 4 body equations

THE RESTRICTED HILL FOUR-BODY PROBLEM WITH APPLICATIONS TO THE EARTH–MOON–SUN SYSTEM

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Point-masses, with space-craft orbiting binary asteroids in mutual orbit about a massive sun.

Hill's Lunar 4 body equations

[Scheeres, 1998]

$$\begin{aligned} x'' - 2(1+m)y' &= V_x, \\ y'' + 2(1+m)x' &= V_y, \\ z'' &= V_z, \end{aligned} \tag{55}$$

where

$$\begin{aligned} V(x, y, z, \tau; \nu, m) \;=\; & \tfrac{1}{2}\left(1 + 2m + \tfrac{3}{2}m^2\right)(x^2 + y^2) - \tfrac{1}{2}m^2 z^2 \\ & + \tfrac{3}{4}m^2((x^2 - y^2)\cos 2\tau - 2xy \sin 2\tau) \\ & + \frac{m^2}{a_0^3}\left[\frac{1-\nu}{R_{1-\nu}} + \frac{\nu}{R_\nu}\right] \end{aligned} \tag{56}$$

and where

$$R_{1-\nu} = \sqrt{[x + \nu(1 + \bar{\xi})]^2 + [y + \nu\bar{\eta}]^2 + z^2}, \tag{57}$$

$$R_\nu = \sqrt{[x - (1-\nu)(1 + \bar{\xi})]^2 + [y - (1-\nu)\bar{\eta}]^2 + z^2}. \tag{58}$$

These equations are periodic in τ with period π. They contain the two parameters ν and m. If $m \to 0$ then the equations of motion for the restricted three-body problem are recovered. Thus Equations (55) are a generalization of both the restricted three-body problem and the Hill equations of motion.

Hetrogenenous system:

Combining different types of processors, accelerators, and specialized hardware to work together.

- CPU
- GPU
- FPGA

Hetrogeneous system: Why?

Performance Boost:
- Leveraging specialized processors for specific tasks leads to enhanced overall performance.

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Energy Efficiency:
- Optimal utilization of resources, reducing power consumption.

Parallel Processing
- Simultaneous execution of tasks, accelerating complex computations.

Heterogenous system: considerations

Programming complexity usually is a barrier, decision about which code would run where (calculating overheads and data "movements" across diverse hardware)

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But....

Heterogeneous computing “done right” is a powerful paradigm for improving performance and efficiency in a wide range of applications.

Heterogenous system: so far

Different parts of PSM were rewritten in VHDL to be synthetized in hardware to study the behavior and the performance of the new dedicated hardware

1. Intel Cyclone V for preliminary testing
   - CPU/FPGA on the same silicon, handy memory sharing but limited FPGA logic elements)
   - Acceptable number of logic elements but somewhat complex memory sharing via PCIe
   - Limited number of hardware wires to submit the large number of inputs/arrays required by an n-body problem

Heterogenous system: so far

Different parts of PSM were rewritten in VHDL to be synthetized in hardware to study the behavior and the performance of the new dedicated hardware

1. Intel Cyclone V for preliminary testing
2. Exploiting the heterogeneous framework oneAPI (by Intel) to bridge CPU/FPGA (and also GPU)
   - No need to struggle with memory mapping
   - No need to port to VHDL, C++ code

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but.....

Heterogenous system: challenges (so far)

- oneAPI support (getting better and better) was/is somewhat limited for different hardware (Cyclone family)
- Machine setup/libraries/dependencies quite complex (not fully supported yet by Linux)
- Several errors/difficult debugging due to limited documentation and knowledge base

Heterogenous system: Looking ahead

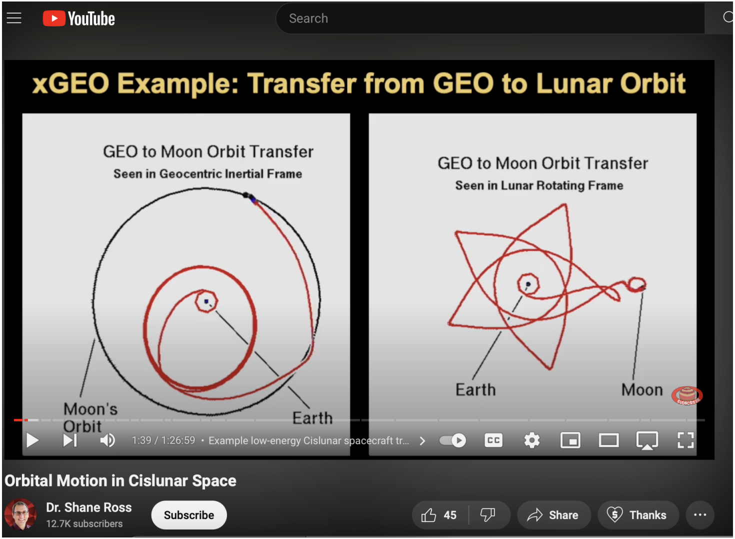
- Finalizing oneAPI approach for PSM/n-body problems so that it can seamlessly/easily compile on a diverse hardware; FPGA+CPU to start with (GPU in the future)/
- online n-body webapp that is hardware accelerated with a dedicated performance benchmark.

Heterogenous system: Goal

Orbital Motion!

Heterogenous system: Goal

Orbital Motion!



from https://www.youtube.com/watch?v=gpXAACF5eOI